11th European Fluid Mechanics Conference: Seville Sept. 12-16, 2016

Eulerian and Lagrangian Statistics in Fourier-reduced Navier Stokes equations

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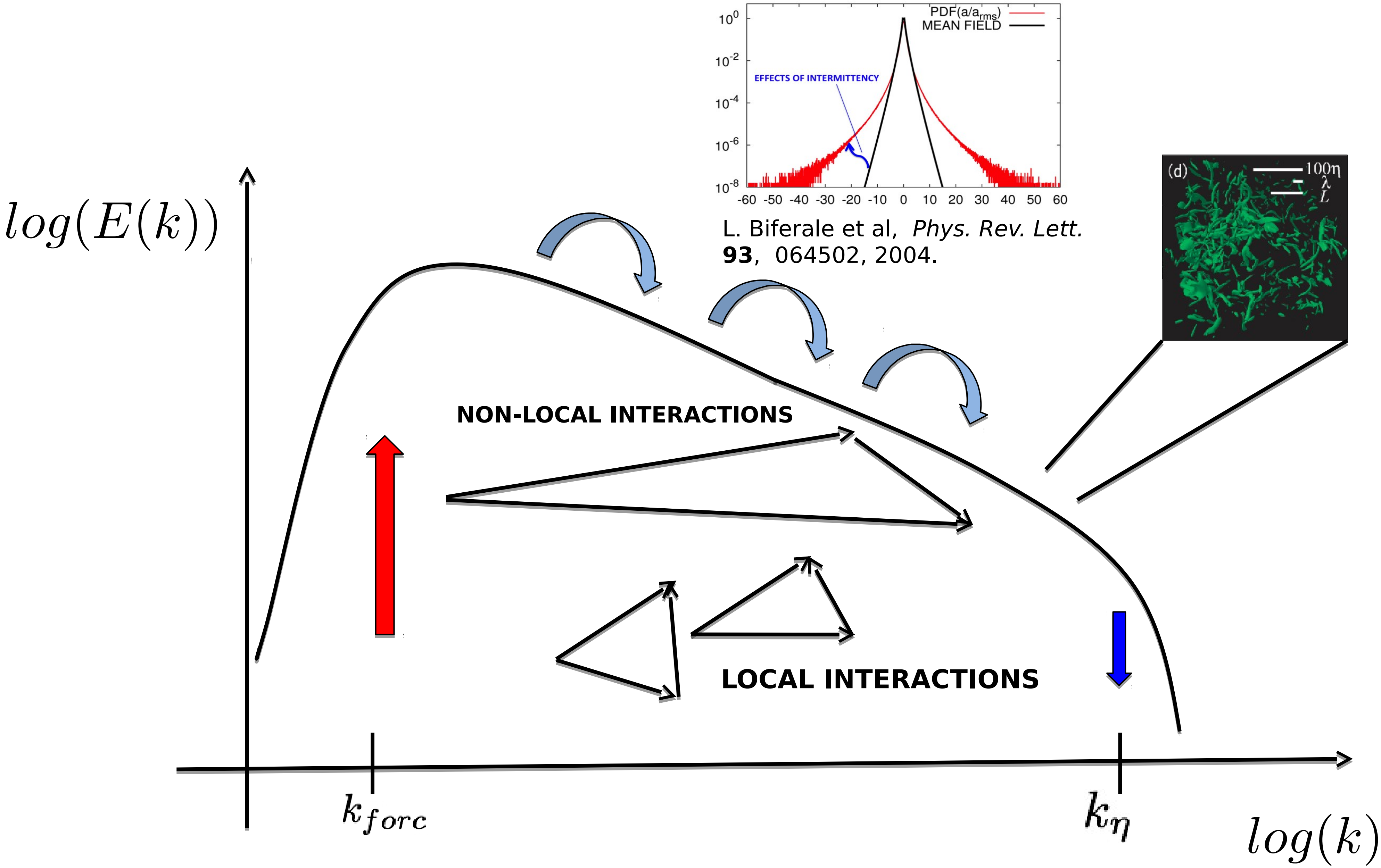
EXPERIMENTS IN-SILICO:
CAN WE ASK QUESTIONS ABOUT THE ENERGY TRANSFER EVENTS
(BOTH TYPICAL AND EXTREME)
BY DECIMATING INTERACTIONS IN THE NON LINEAR TERM?

Luca Biferale
Alessandra Lanotte
Samriddhi Sankar Ray
Akshay Bhatnagar

Ref.:
- M. Buzzicotti, L. Biferale, U. Frisch, and S. S. Ray, **Phys. Rev. E 93, 033109 (2016)**.
- A.S. Lanotte, S. K. Malapaka, and L. Biferale, **Eur. Phys. J. E 39, 49 (2016)**.
- M. Buzzicotti, A. Bhatnagar, L. Biferale, A.S. Lanotte and S.S. Ray. **Submitted to NJP, (2016)**.

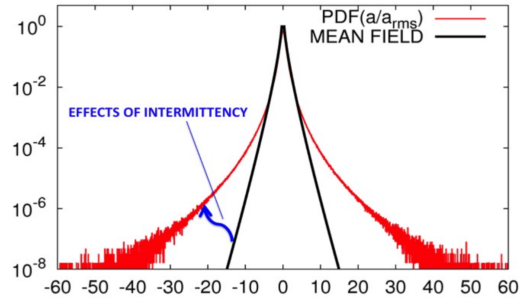
ERC Advanced Grant (N. 339032) "NewTURB"



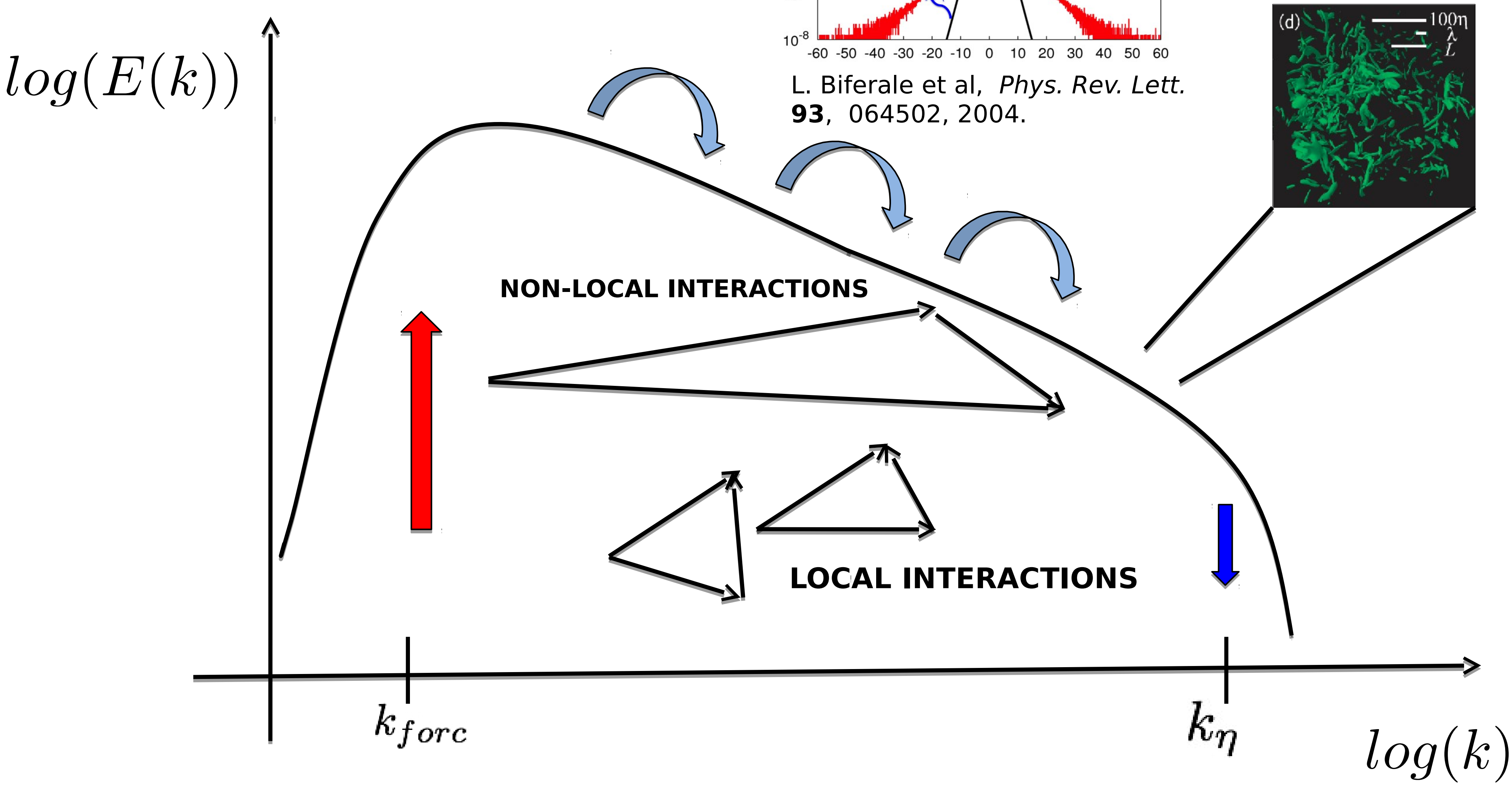
L. Biferale et al, *Phys. Rev. Lett.* **93**, 064502, 2004.

$$\partial_t \hat{u}_n(\mathbf{k},t) + \left(\delta_{nm} - \frac{k_n k_m}{|k|^2}\right) NL_m(\mathbf{k},t) = -\nu|\mathbf{k}|^2 \hat{u}_n(\mathbf{k},t) + \hat{f}_n(\mathbf{k},t)$$

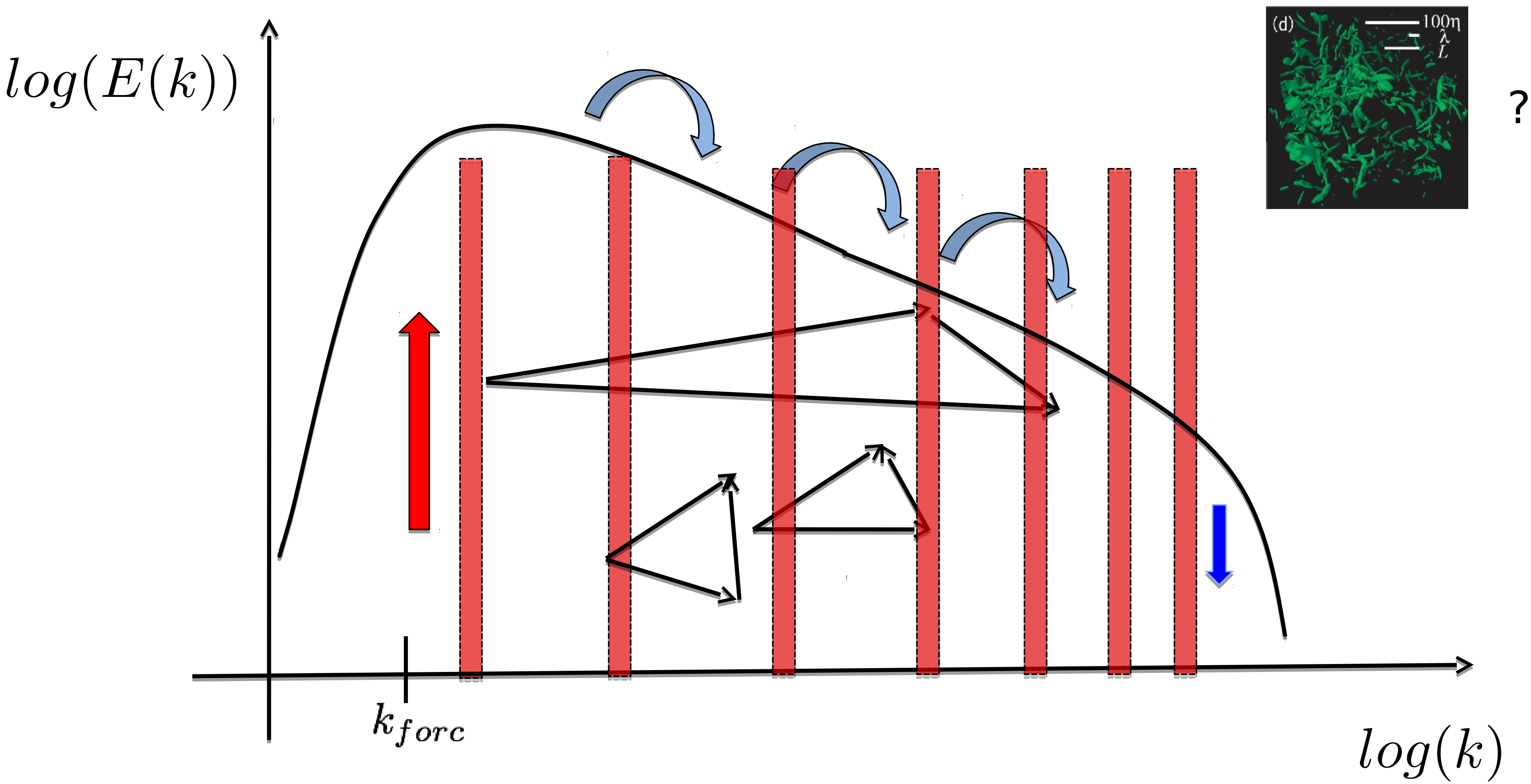
$$NL_m(\mathbf{k},t) = -i \sum_{\mathbf{k}'+\mathbf{k}''=\mathbf{k}} k''_j \hat{u}_m(\mathbf{k}',t)\hat{u}_j(\mathbf{k}'',t)$$



L. Biferale et al, *Phys. Rev. Lett.* **93**, 064502, 2004.

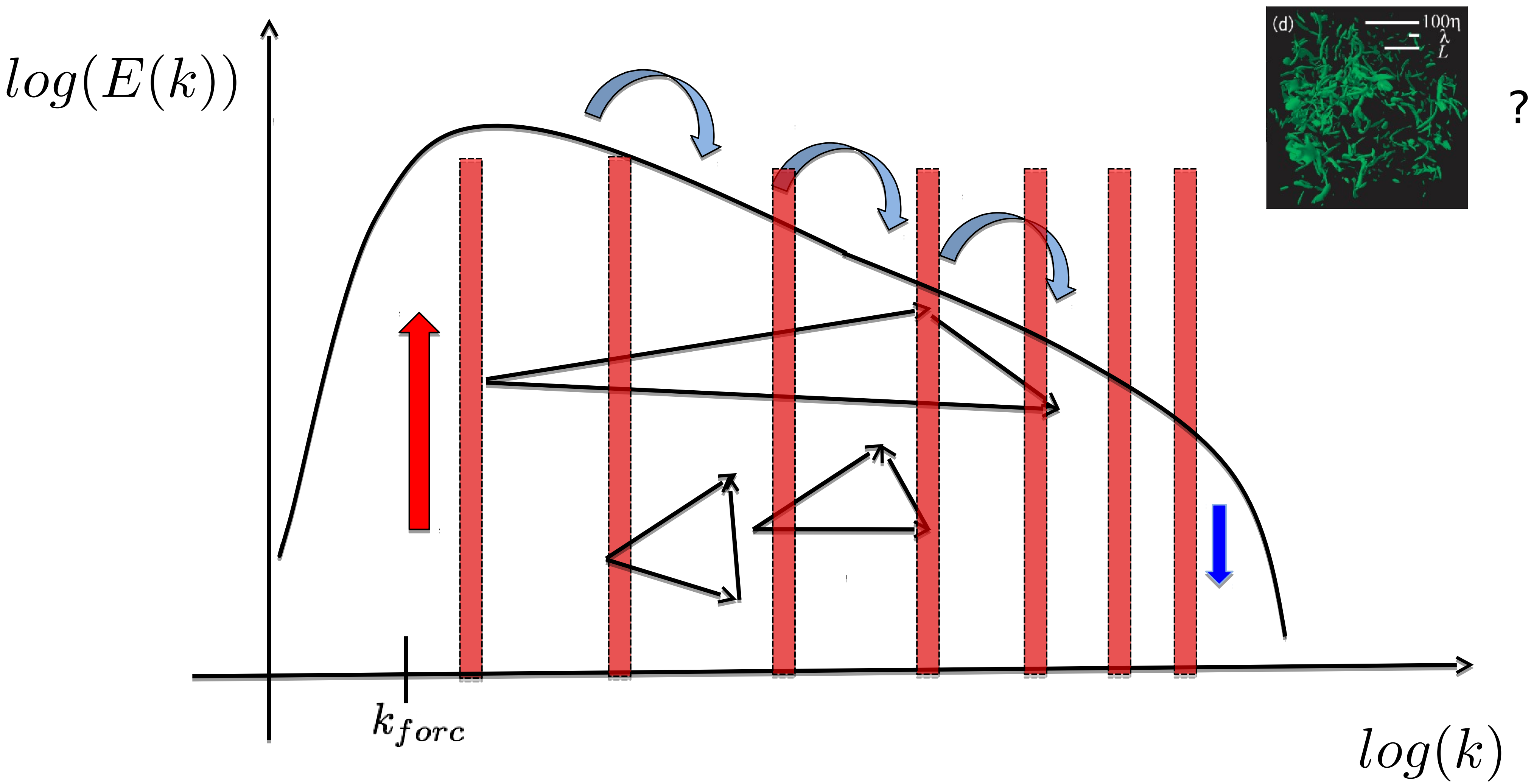


How many (and which) degrees of freedom do we need to preserve the main statistical properties of NS turbulence?



$$u^D(x,t) = P(x)u(x,t) = \sum_{k \in Z^3} e^{ikx}\theta(k)\hat{u}(k,t)$$

$$\theta(k) = \begin{cases} 1 & with \;\; probability \;\; P(k) \\ 0 & with \;\; probability \;\; 1 - P(k) \end{cases}$$



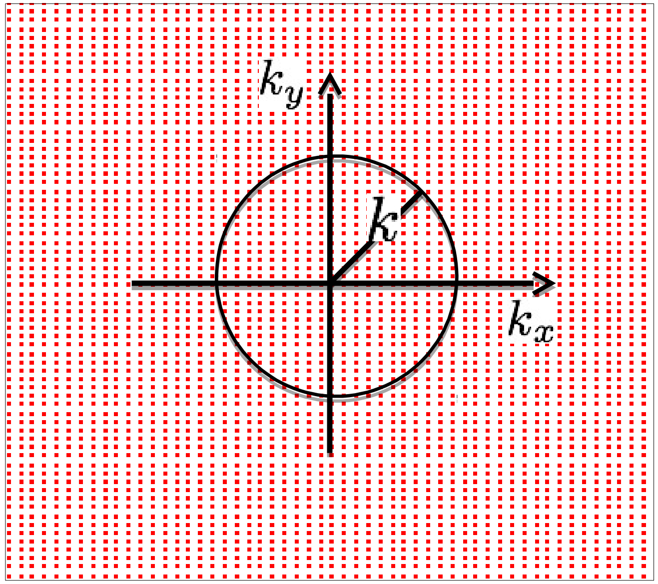
$$\begin{cases} P(k) & = k^{(D-3)} \\ P(k) & = \alpha \end{cases}$$

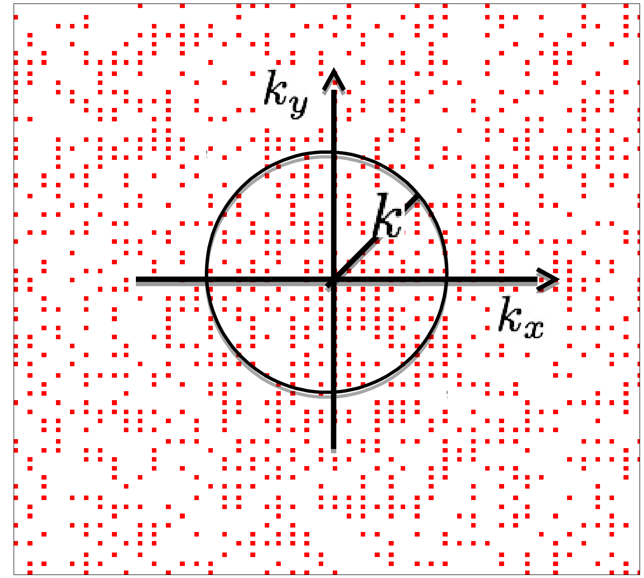
→ Fractal decimation

→ Homogeneous decimation

$$\begin{cases} \#_{dof} = & \int_0^k P(k')\, dk' \propto k^D \\ \#_{dof} = & \int_0^k P(k')\, dk' \propto \alpha k^3 \end{cases}$$

SELF-SIMILAR GALERKIN TRUNCATION

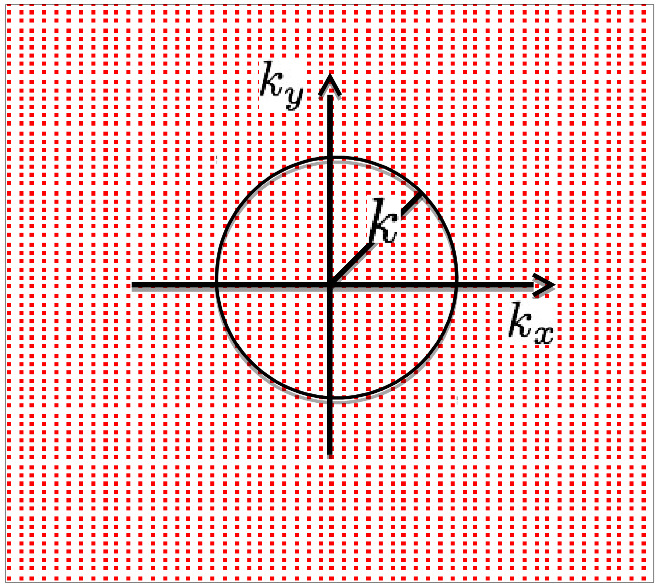


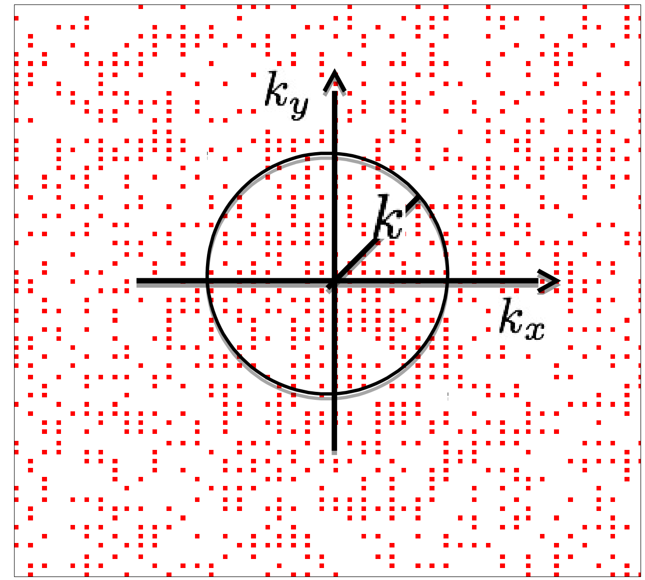


HOMOGENEOUS & ISOTROPIC & SELF-SIMILAR (NO EXTERNAL SCALES)
ENERGY & HELICITY INVISCID INVARIANTS
REAL PDE (INFINITE NUMBER OF DEGREES OF FREEDOM)

$$\partial_t \hat{u}_n(\mathbf{k},t) + \left(\delta_{nm} - \frac{k_n k_m}{|k|^2}\right) NL_m(\mathbf{k},t) = -\nu|\mathbf{k}|^2 \hat{u}_n(\mathbf{k},t) + \hat{f}_n(\mathbf{k},t)\,; \quad \hat{u}(\mathbf{k},t) \to P_D \hat{u}(\mathbf{k},t)$$

SELF-SIMILAR GALERKIN TRUNCATION





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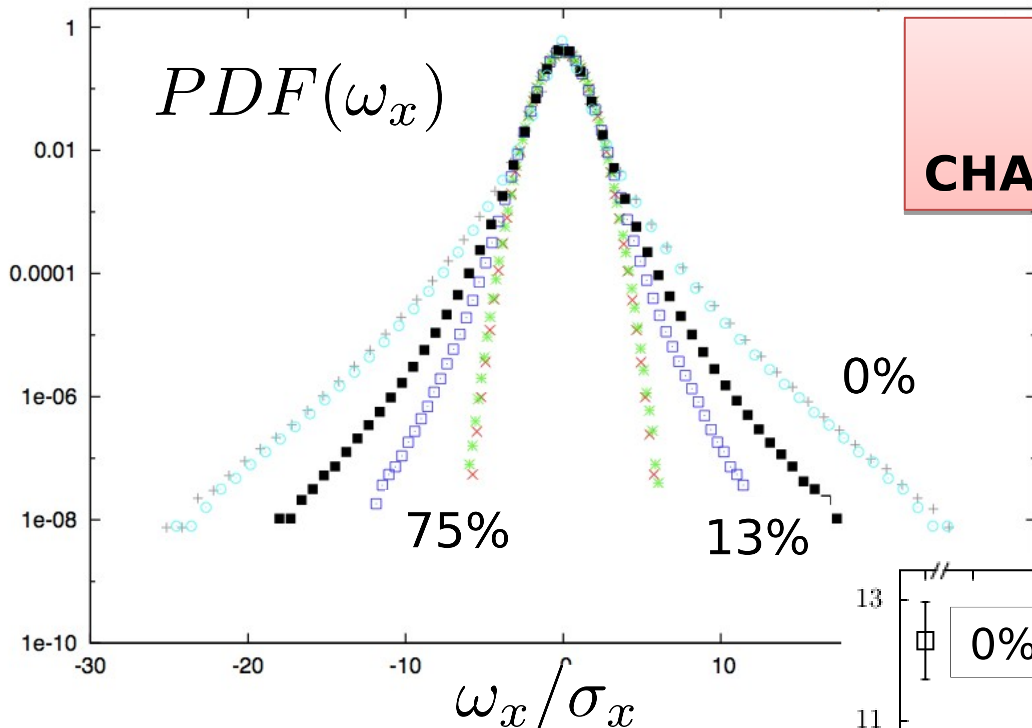
$$\partial_t \hat{u}_n^D(\mathbf{k},t) + \left(\delta_{nm} - \frac{k_n k_m}{|k|^2}\right) P_D N L_m^D(\mathbf{k},t) = -\nu |\mathbf{k}|^2 \hat{u}_n^D(\mathbf{k},t) + \hat{f}_n^D(\mathbf{k},t)$$

PRL **115,** 264502 (2015) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2015

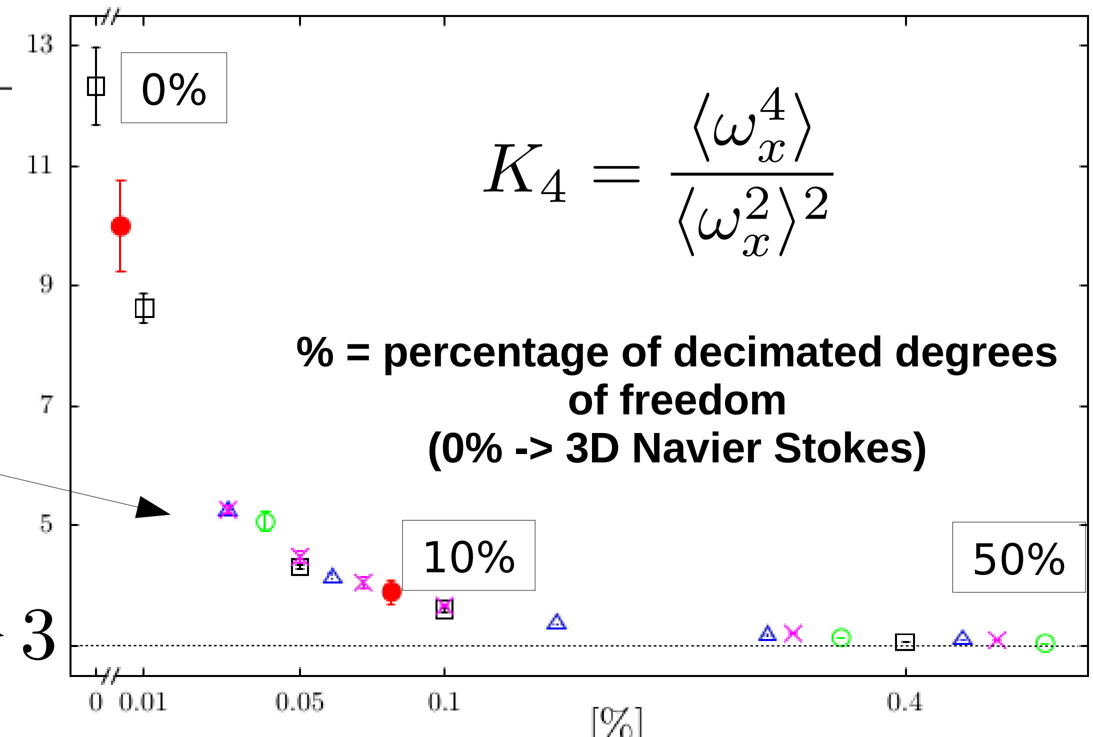
Turbulence on a Fractal Fourier Set

Alessandra S. Lanotte,[1,*] Roberto Benzi,[2] Shiva K. Malapaka,[2,3] Federico Toschi,[4] and Luca Biferale[2]

PDF OF VORTICITY AT CHANGING FRACTAL DIMENSION



$$\omega = \nabla \times \mathbf{v}$$

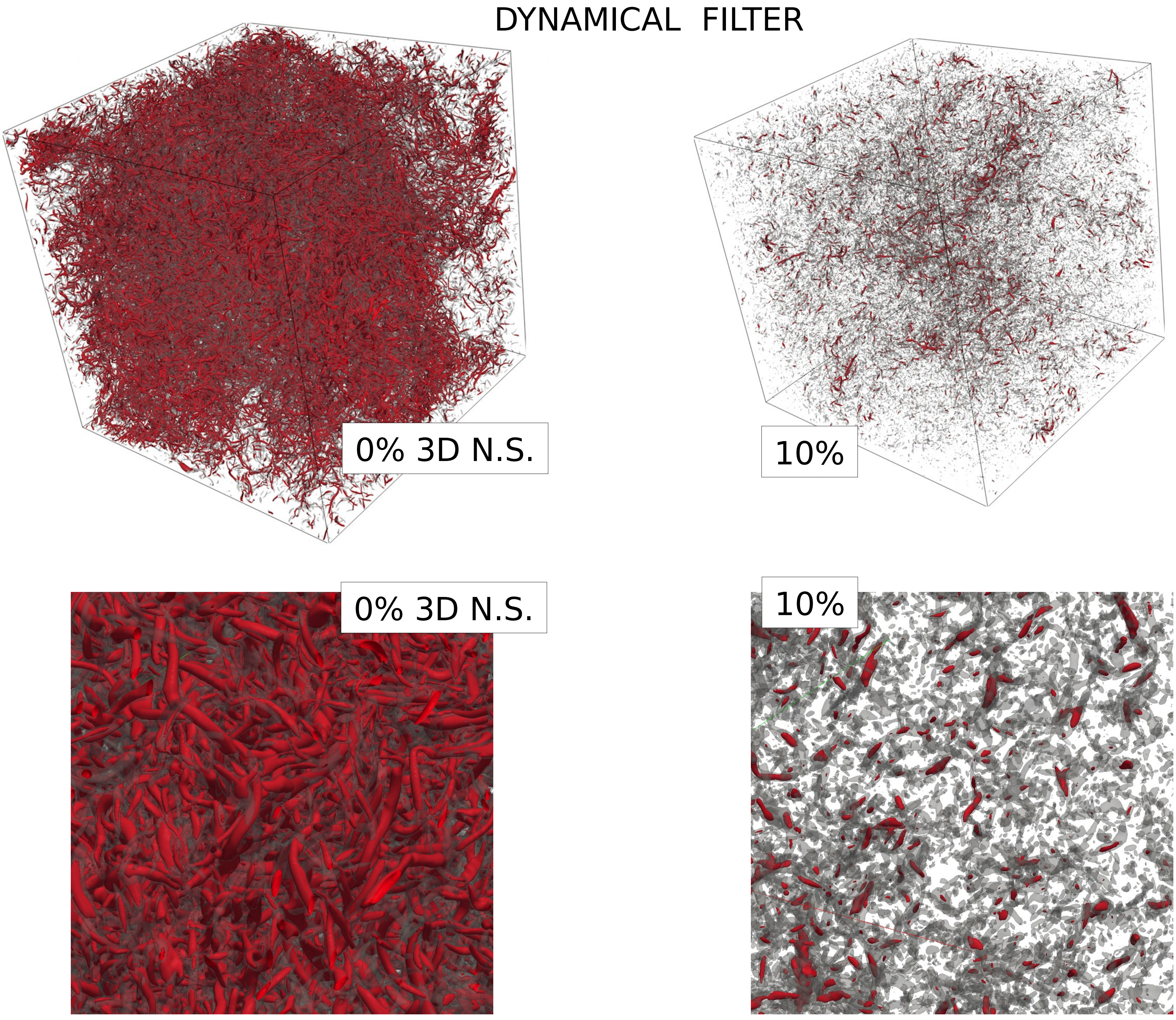


$$K_4 = \frac{\langle \omega_x^4 \rangle}{\langle \omega_x^2 \rangle^2}$$

% = percentage of decimated degrees of freedom (0% -> 3D Navier Stokes)

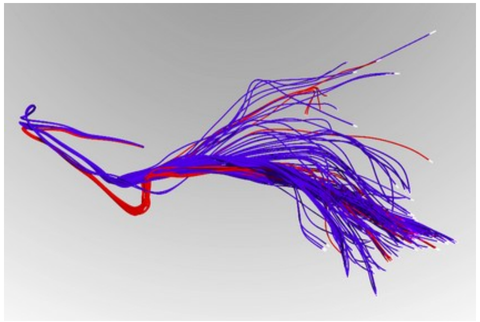
Different point styles are for different Reynolds and decimation protocol

K_4 *Gaussian Distribution* → 3

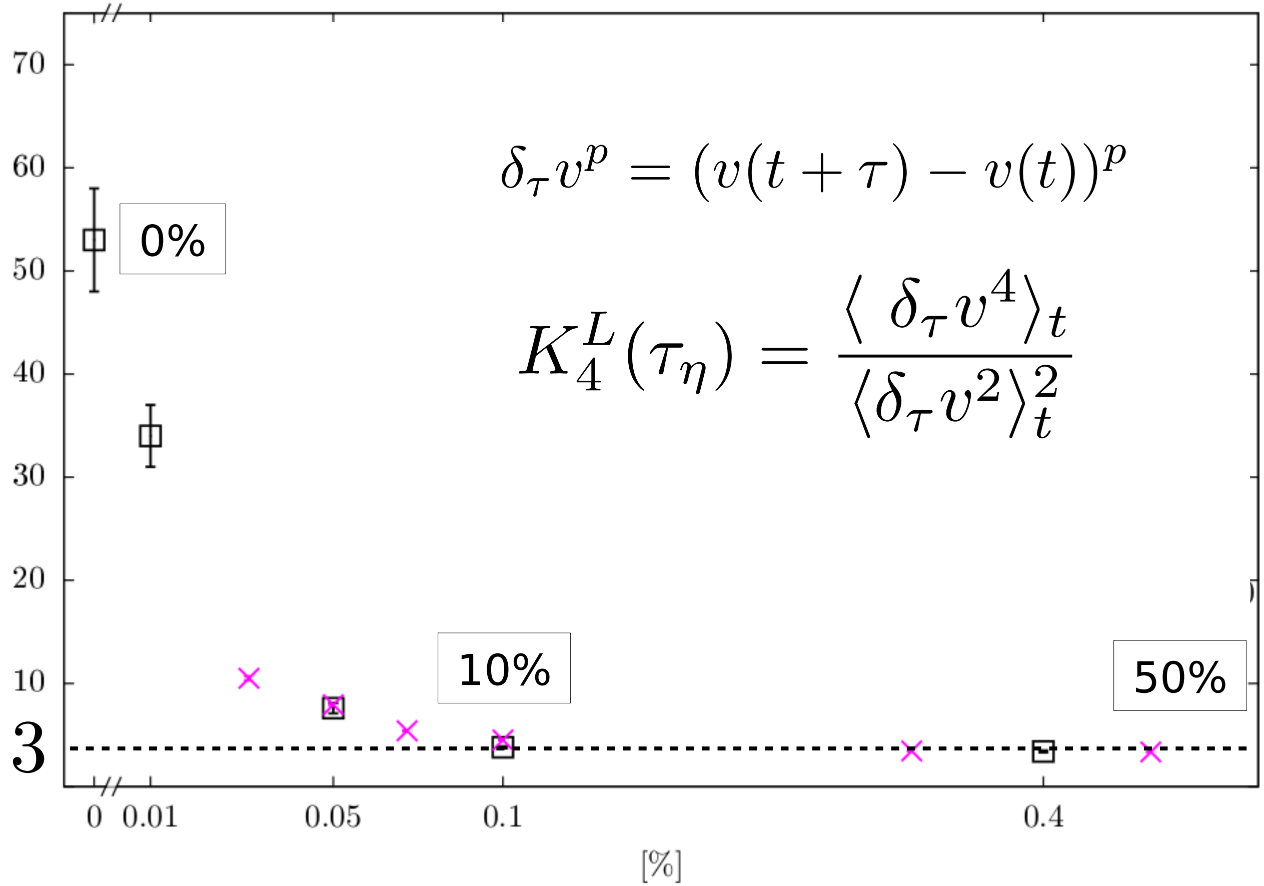

DYNAMICAL FILTER
0% 3D N.S.
10%
0% 3D N.S.
10%

Lagrangian Intermittency

$$\begin{cases} \frac{\partial \mathbf{x}(t)}{\partial t} = \mathbf{v}^D(t) = \mathbf{u}^D(\mathbf{x}(t), t) \\ \partial_t \mathbf{u}^D = P(\mathbf{x})(\mathbf{u}^D \nabla)\mathbf{u}^D + \Delta \mathbf{u}^D + \mathbf{f}^D \end{cases}$$



K_4 *Gaussian Distribution*



Bridge Relation

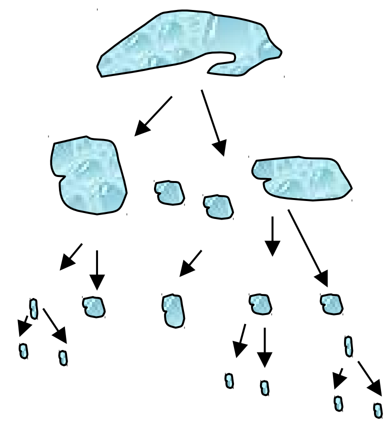
$$\delta_r u = (u(x+r) - u(x))$$

$$S_p(r) = \langle \delta_r u^p \rangle \sim r^{\zeta_E(p)}$$

$$\delta_\tau v = (v(t+\tau) - v(t))$$

$$S_p(\tau) = \langle \delta_\tau v^p \rangle \sim \tau^{\zeta_L(p)}$$

In the **Multifractal terminology**: $\delta_r u \sim r^h; \; P(h) \sim r^{3-D(h)}$



$$\zeta_E(p) = \inf_h \left[hp + 3 - D(h)\right]$$

$$\zeta_L(p) = \inf_h \left[\frac{hp + 3 - D(h)}{1-h}\right]$$

Relation between Lagrangian and Eulerian increments

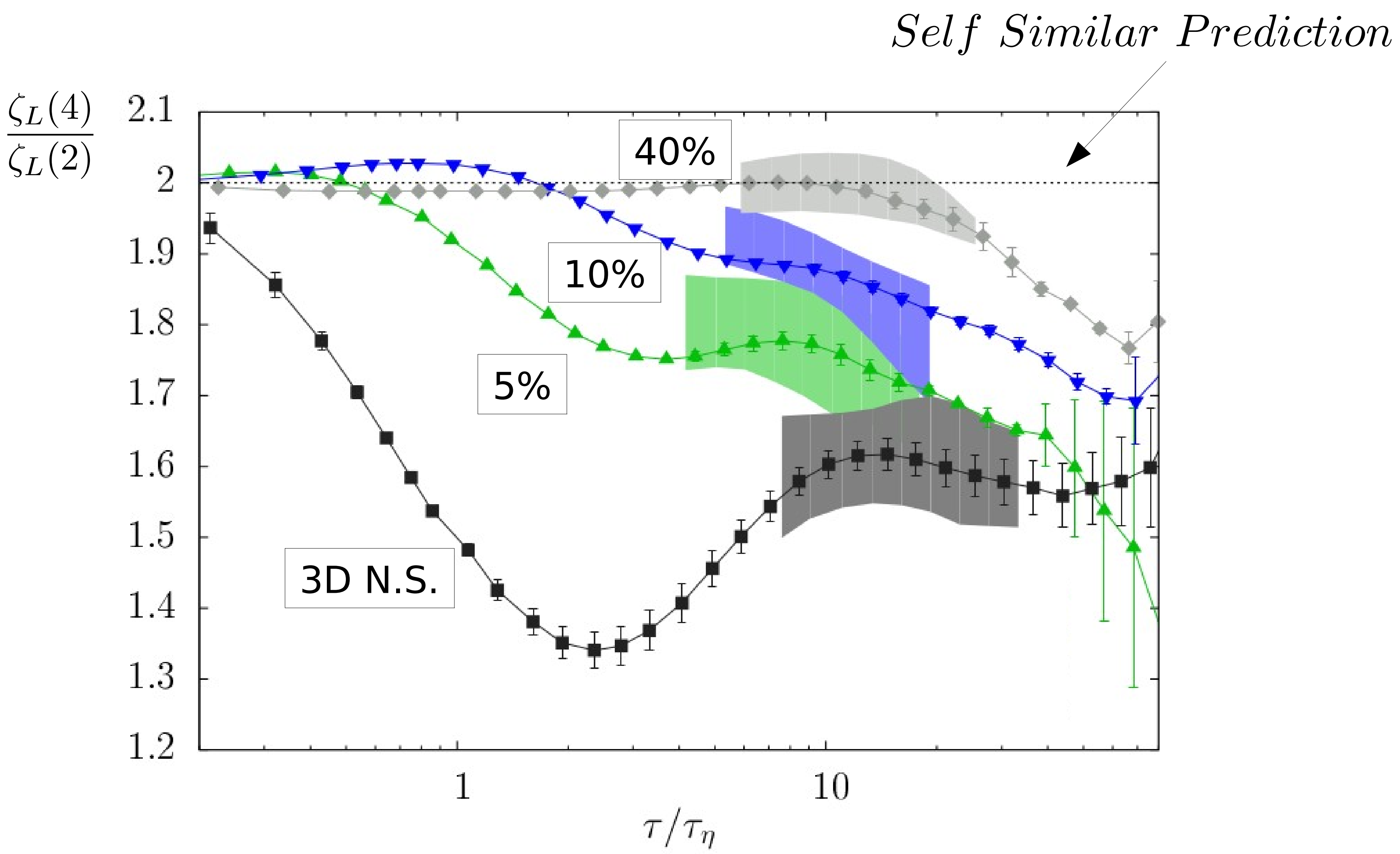
-) Boffetta, Mazzino, Vulpiani, Journal of Physics A, 41(36), 363001 - (2008)

-) A. Arnèodo et al., Phys. Rev. Lett. 100, 254504 - 2008

-) F.G. Schmitt, Physica A 368 377, 386 - 2006

Lagrangian Intermittency in fractally decimated Turbulence

$$S_p(\tau) \sim \tau^{\zeta_L(p)} \rightarrow \zeta_L(p) = \frac{\partial\left(log(S_p(\tau))\right)}{\partial log(\tau)}$$



Conclusions

+ CORRECTION TO FLUCTUATIONS: HUGE. SMALL SCALE VORTICITY IS STRONGLY SENSITIVE TO DECIMATION. “COHERENT” SMALL-SCALE STRUCTURES FEEL GLOBAL CORRELATIONS ACROSS SCALES IN FOURIER.

+ HOW TO BRING INTERMITTENCY BACK TO DECIMATED NS EQUATIONS?

+ THE INTERMITTENCY DISAPPEARS ALSO IN THE LAGRANGIAN STATISTIC, FOLLOWING THE BRIDGE RELATION.

- WE STILL MISS A CLEAR DEFINITION OF INTERMITTENCY IN FOURIER SPACE

Ref.:

- M. Buzzicotti, L. Biferale, U. Frisch, and S. S. Ray, **Phys. Rev. E 93, 033109 (2016)**.
- A.S. Lanotte, S. K. Malapaka, and L. Biferale, **Eur. Phys. J. E 39, 49 (2016)**.
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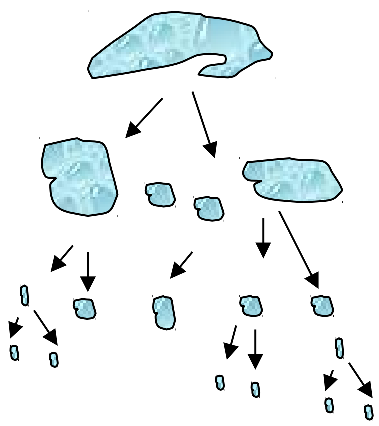
Bridge Relation

$$\delta_r u = (u(x+r) - u(x)) \qquad \delta_\tau v = (v(t+\tau) - v(t))$$

$$S_p(r) = \langle \delta_r u^p \rangle \sim r^{\zeta_E(p)} \qquad S_p(\tau) = \langle \delta_\tau v^p \rangle \sim \tau^{\zeta_L(p)}$$

$$\delta_r u \sim r^h; \;\; P(h) \sim r^{3-D(h)}$$



$$S_p(r) \sim \int_I r^{3-D(h)} r^{hp} dh \xrightarrow{r\to 0}$$

$$\tau_r \sim \frac{r}{\delta_r u} \to \tau_r \sim r^{1-h}$$

$$S_p(\tau) \sim \int_I \tau^{(3-D(h))/(1-h)} \tau^{hp/(1-h)} dh \xrightarrow{\tau\to 0}$$

$$\zeta_E(p) = \inf_h \left[hp + 3 - D(h)\right]$$

$$\zeta_L(p) = \inf_h \left[\frac{hp + 3 - D(h)}{1-h}\right]$$

Relation between Lagrangian and Eulerian increments