

Extreme events and phases synchronization in Navier-Stokes equations and in non-linear models

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Motivation:

Understanding the robustness of the energy transfer mechanism driven by singular events.

M. Buzzicotti , B.P. Murray, L. Biferale, M.D. Bustamante. The European Physical Journal E, 39(3), 1-9, (2016).

Turbulent Flows

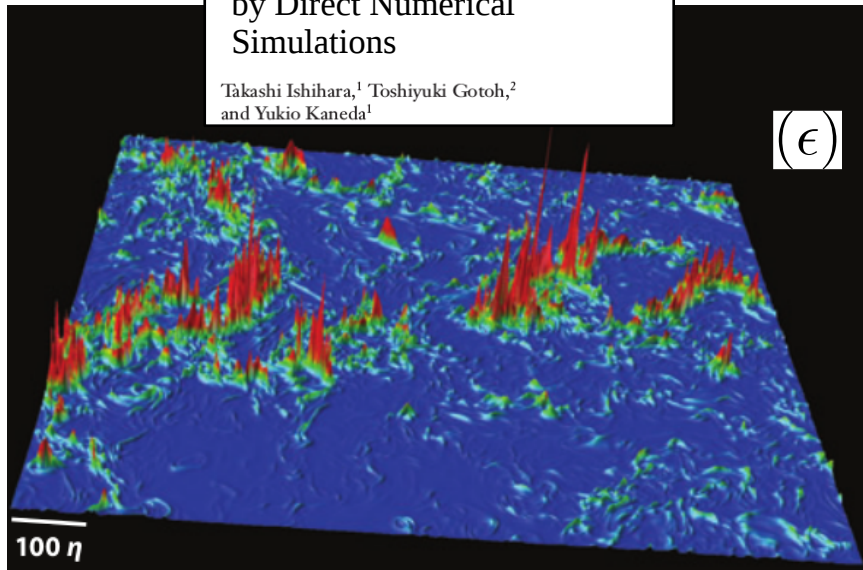
$$\left\{ \begin{aligned} \partial_t \bar{u}(\bar{x}, t) + \bar{u}(\bar{x}, t) \cdot \nabla_{\bar{x}} \bar{u}(\bar{x}, t) &= -\nabla_{\bar{x}} p(\bar{x}, t) + \nu \Delta_{\bar{x}} \bar{u}(\bar{x}, t) + \bar{f}(\bar{x}, t) \\ \nabla_{\bar{x}} \cdot \bar{u}(\bar{x}, t) &= 0 \end{aligned} \right.$$

System **out of equilibrium**

Strongly **not Gaussian energy transfer** from large to small scales

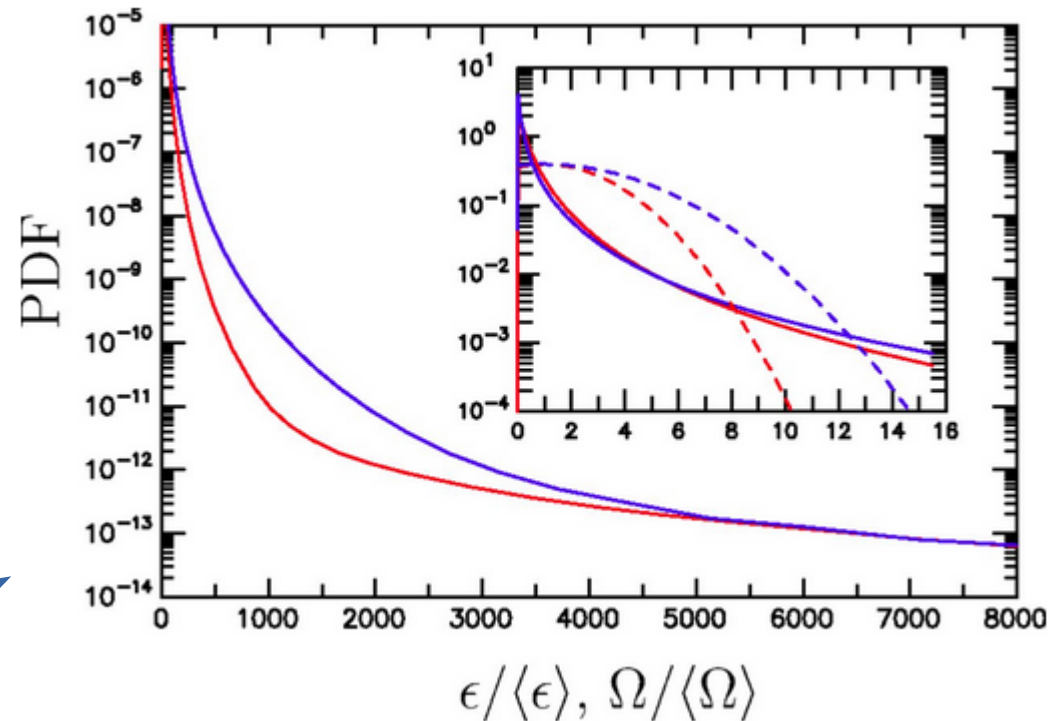
Study of High-Reynolds Number Isotropic Turbulence by Direct Numerical Simulations

Takashi Ishihara,¹ Toshiyuki Gotoh,² and Yukio Kaneda¹



Extreme events in computational turbulence

P. K. Yeung^a, X. M. Zhai^b, and Katepalli R. Sreenivasan^{c,1}

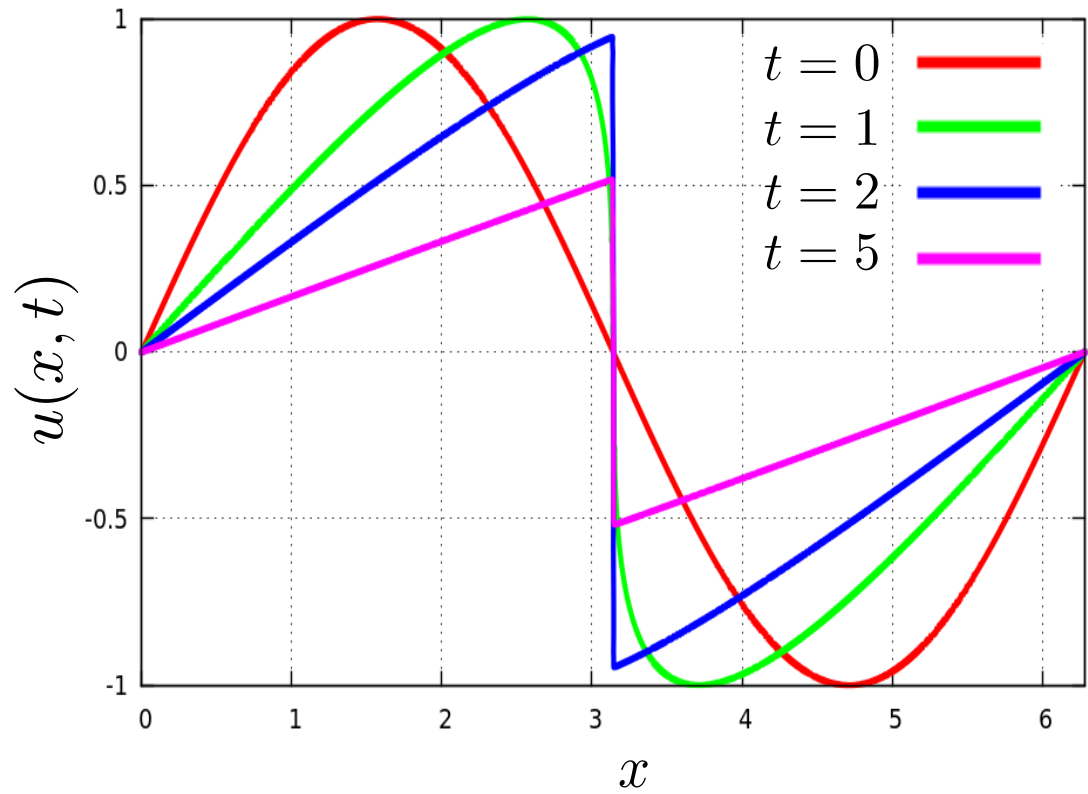


$$N = 4096^3, Re_\lambda \sim 700$$

$$N = 8192^3, Re_\lambda \sim 1300$$

Burgers' equation

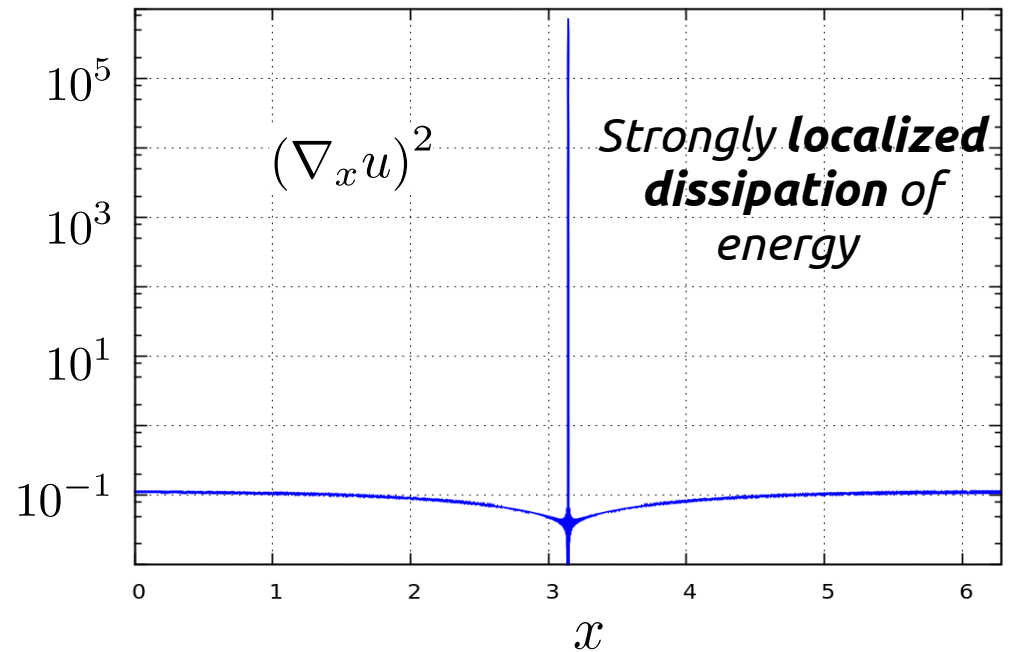
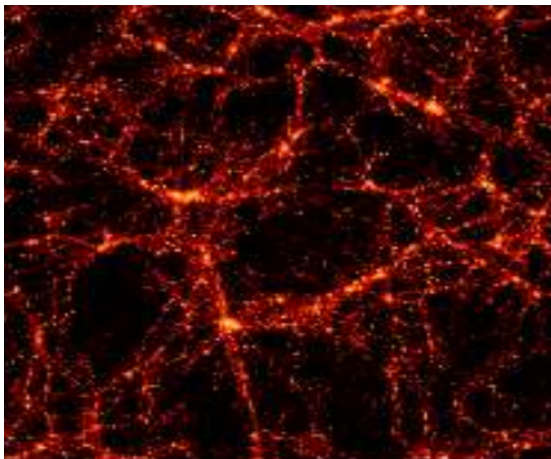
$$\frac{\partial u(x, t)}{\partial t} + u \frac{\partial u(x, t)}{\partial x} = \nu \frac{\partial^2 u(x, t)}{\partial x^2}$$



Kardar - Parisi - Zhang (KZP)

$$\frac{\partial h}{\partial t} - \frac{1}{2} |\nabla h|^2 = \nu \nabla^2 h + F$$

2D Burgers; Cosmological model

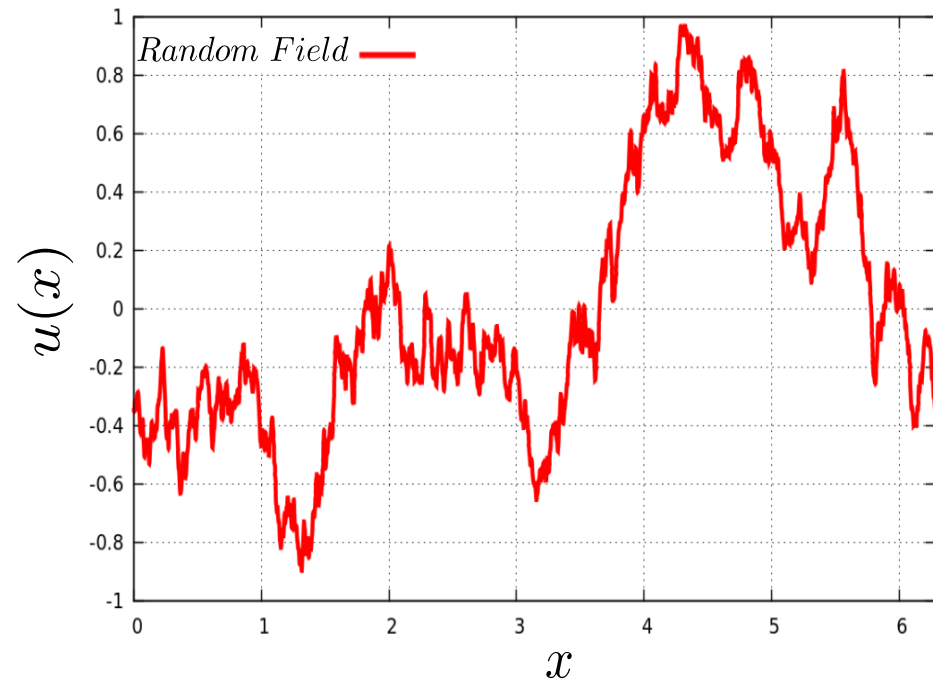
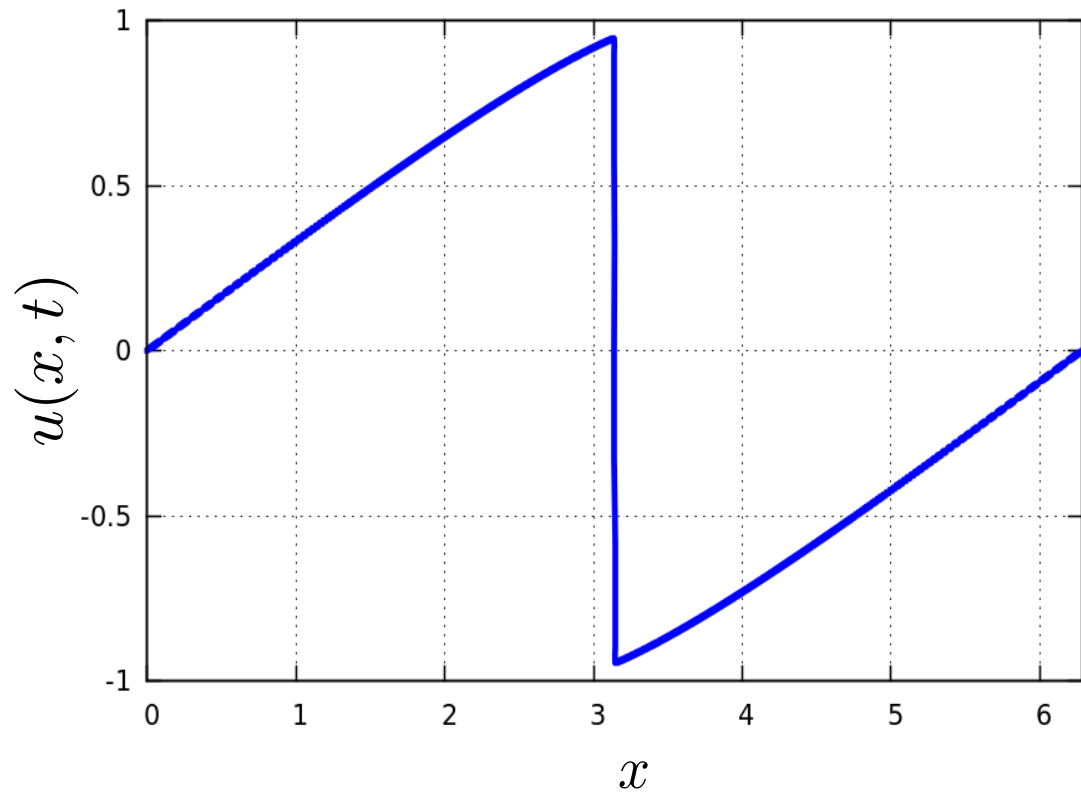
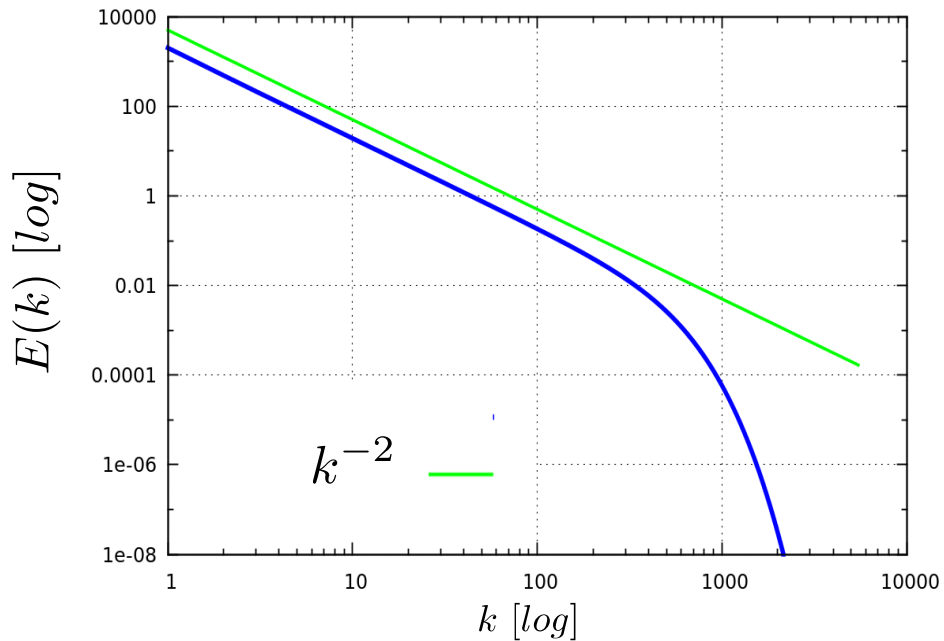


Burgers' equation

$$\frac{\partial u(x, t)}{\partial t} + u \frac{\partial u(x, t)}{\partial x} = \nu \frac{\partial^2 u(x, t)}{\partial x^2}$$

Fourier Space Energy spectrum:

$$E(k) = u(k)u^*(k)$$



**Phases must be the responsible of the singular energy focusing
(small scale shock formation).**

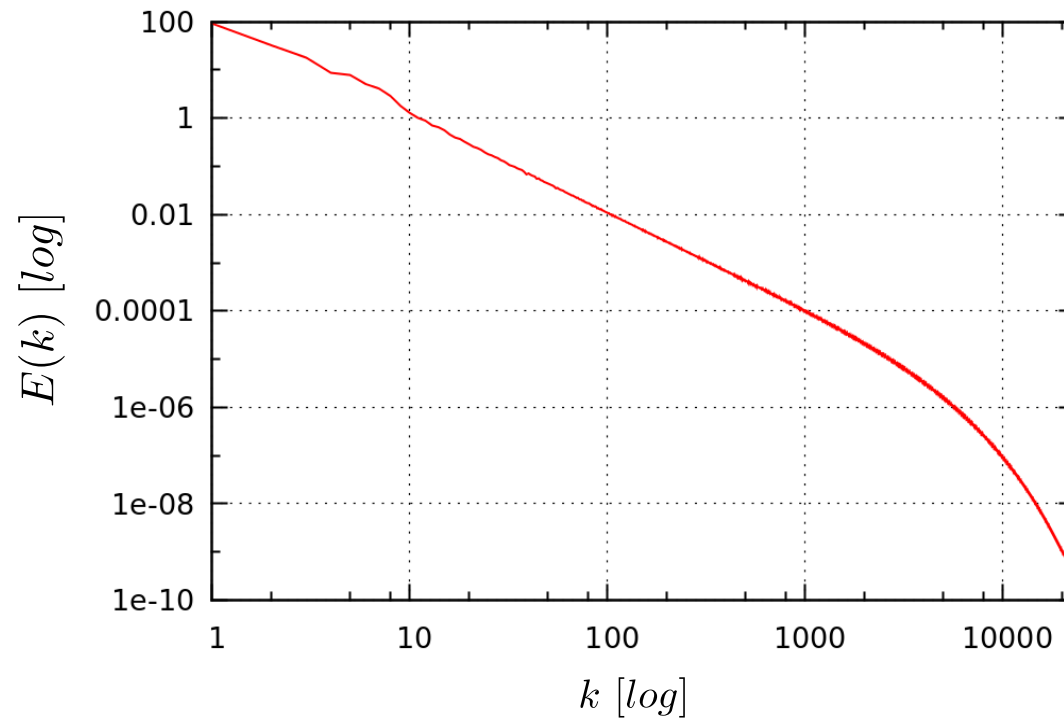
Question: How many degrees of freedom do we need?



Reduce to learn!

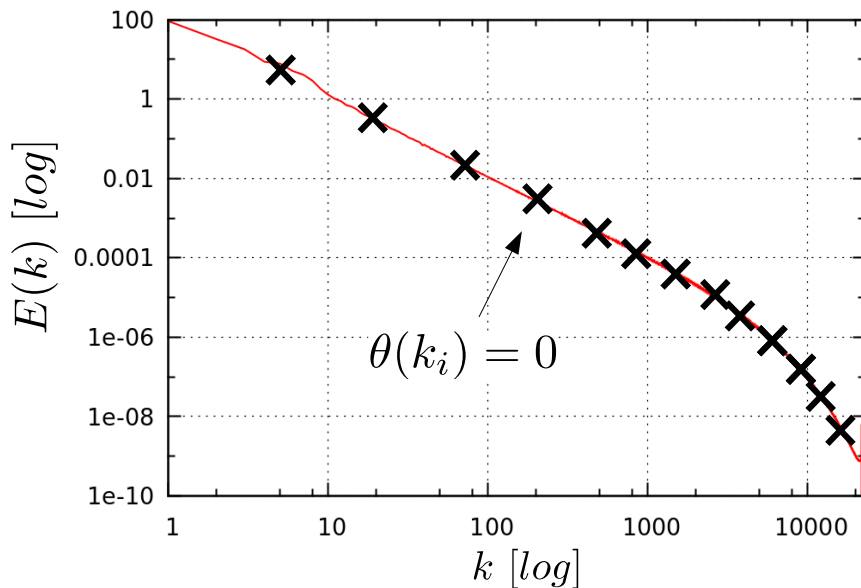
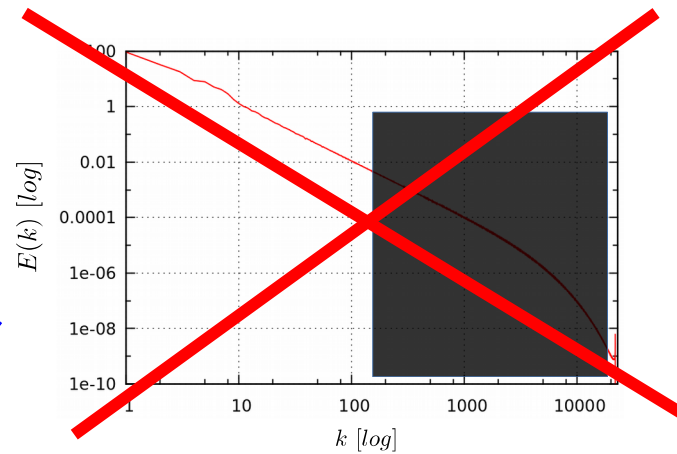
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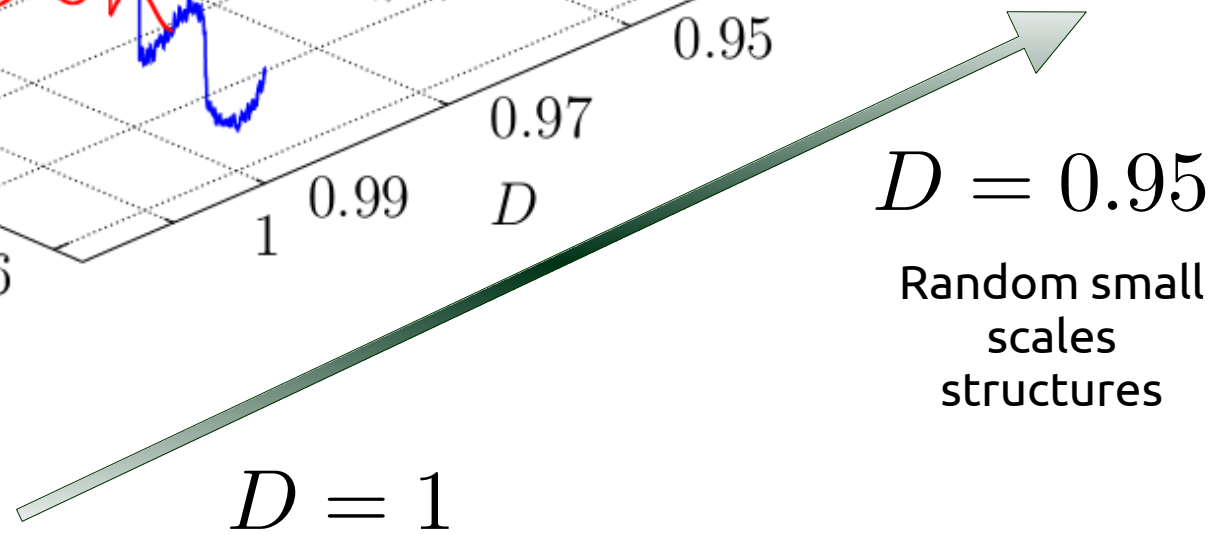
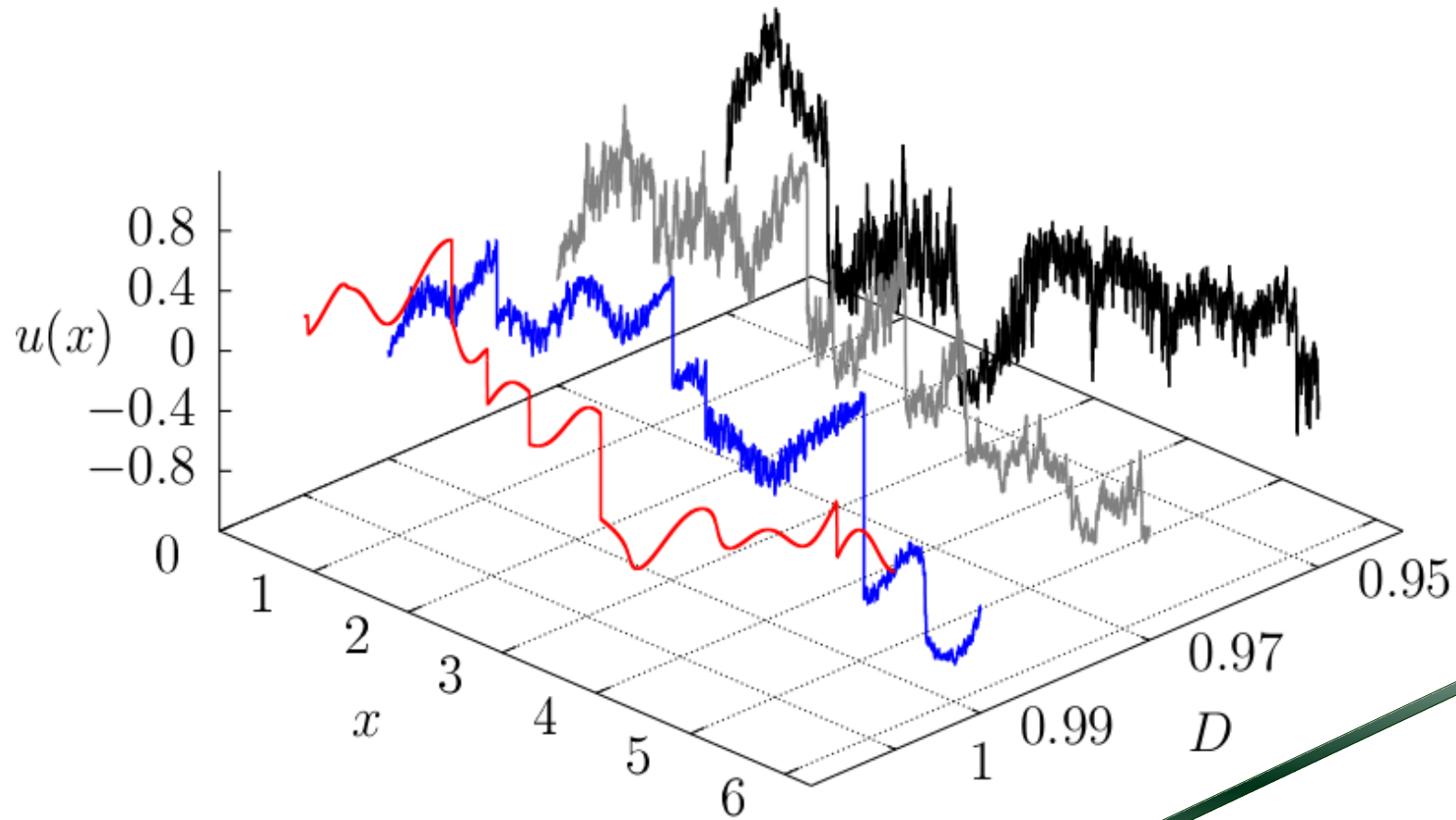
$$v(x, t) = \sum_{k \in \mathbb{Z}} e^{ikx} \theta(k) \hat{u}(k, t)$$

$$\theta(k) = \begin{cases} 1 & \text{with probability } h_k \sim (k/k_0)^{D-1} \\ 0 & \text{with probability } 1 - h_k \end{cases}$$

$$\#_{dof} = \int_0^k \theta(k') dk' \propto k^D$$

D is the system dimension; $0 < D \leq 1$

Real space evolution at changing of fractal dimension:

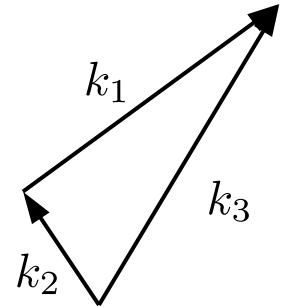


Shock evolution in
energy decaying

Fourier space Burgers' equation + Forcing

Triadic Interactions

$$\frac{d\hat{u}_k}{dt} = -\frac{ik}{2} \sum_{k_1, k_2 \in \mathbb{Z}} \hat{u}_{k_1} \hat{u}_{k_2} \delta_{k_1+k_2, k} - \nu k^2 \hat{u}_k + \hat{F}_k$$



Amplitude – Phase representation:

$$\hat{u}_k = a_k(t) e^{i\phi_k(t)} \text{ where; } a_k(t) = |\hat{u}_k(t)|, \quad \phi_k(t) = \arg \hat{u}_k(t)$$

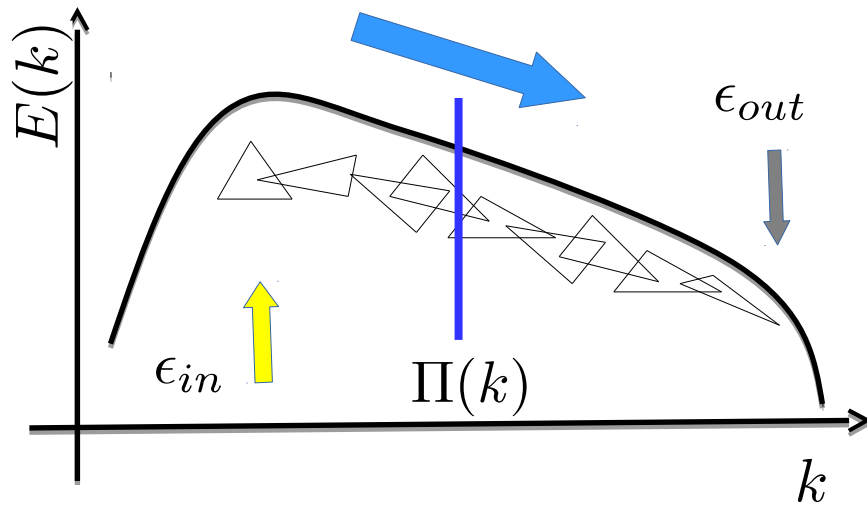
Energy flux towards small scales through k :

$$\Pi(k) = \sum_{k_1=1}^k \sum_{k_3+k_2=k_1}^{\infty} 2k_1 \left\langle a_{k_1} a_{k_2} a_{k_3} \sin(\varphi_{k_1, k_2}^{k_3}) \right\rangle$$

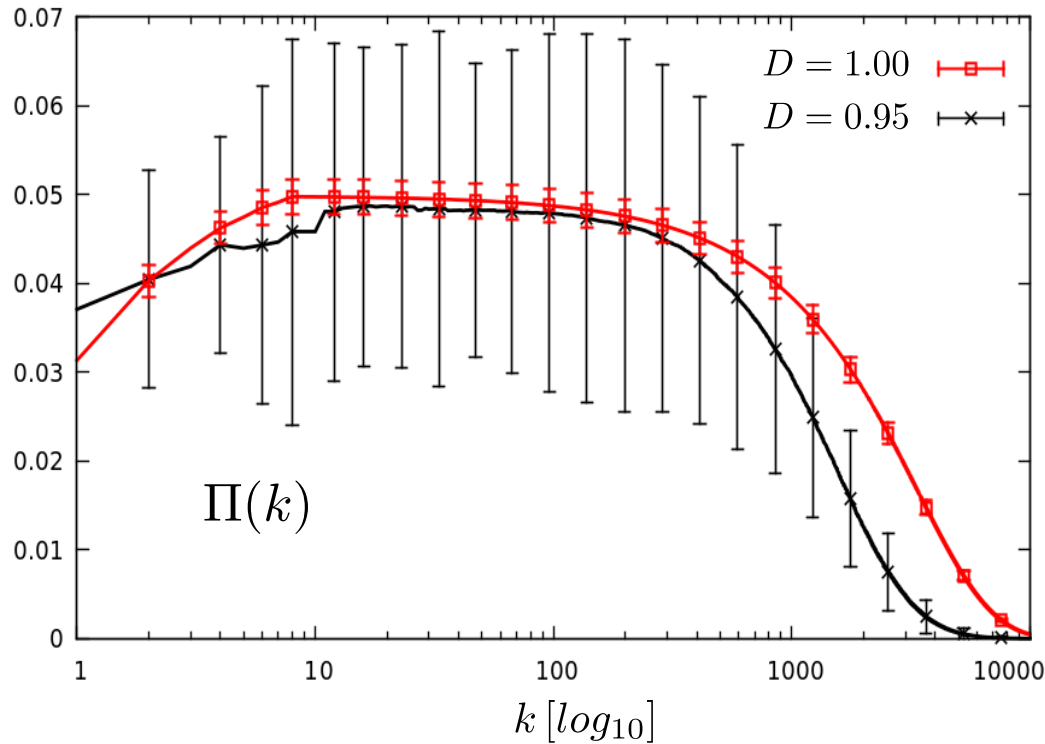
Key degrees of freedom: Triad dynamical phases

$$\varphi_{k_1 k_2}^{k_3}(t) = \phi_{k_1}(t) + \phi_{k_2}(t) - \phi_{k_3}(t), \quad k_1 + k_2 - k_3 = 0$$

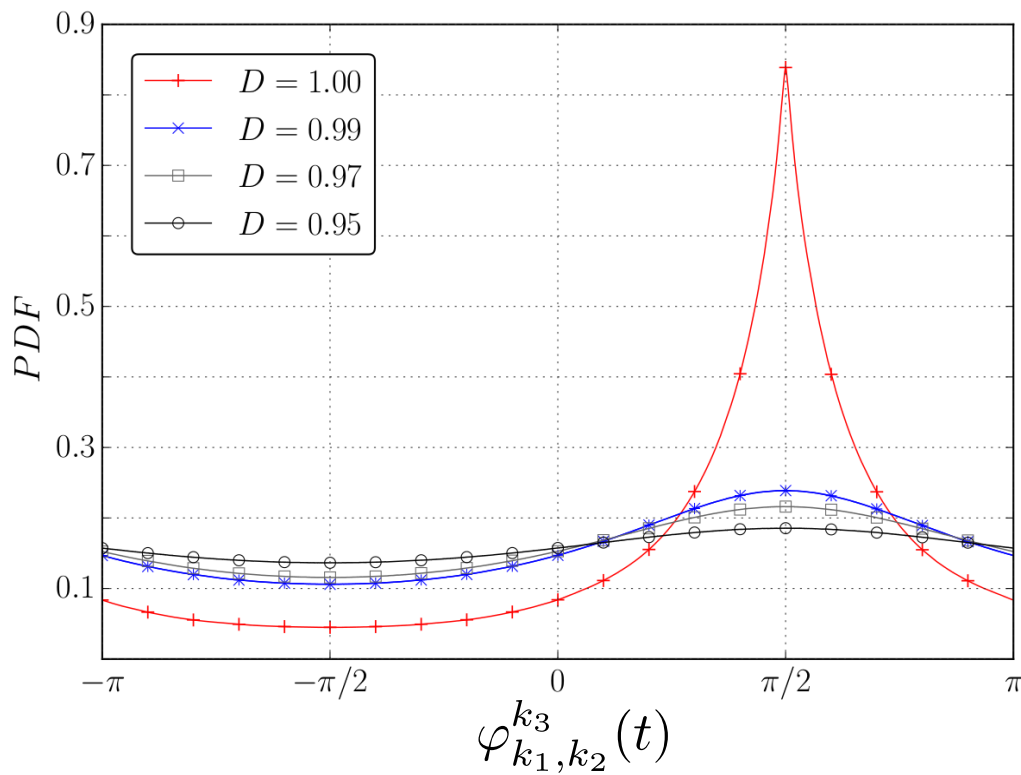
Energy flux towards small scales through k :



$$\Pi(k) = \sum_{k_1=1}^k \sum_{k_3=k+1}^{\infty} 2k_1 \left\langle a_{k_1} a_{k_2} a_{k_3} \sin(\varphi_{k_1, k_2}^{k_3}) \right\rangle$$

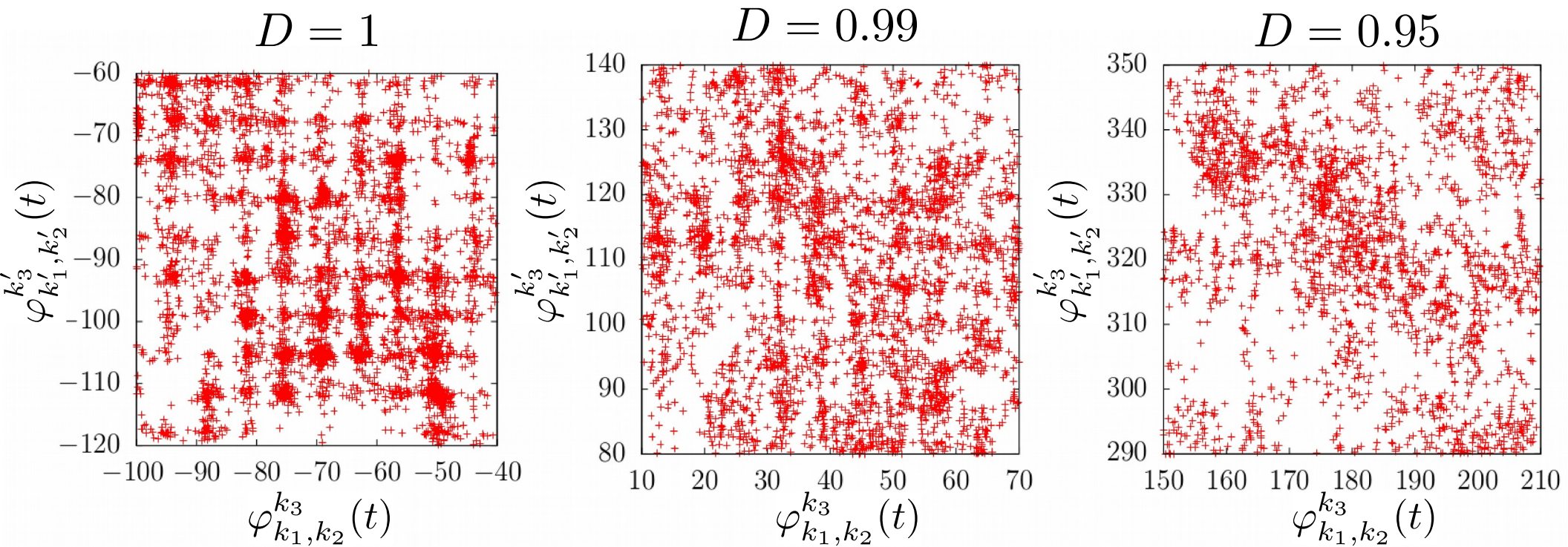


Triad **Phases Align** & **Synchronize** - Efficient Transfers



$$\begin{aligned} \Pi(k) &= \\ &= \sum_{k_1=1}^k \sum_{k_3=k+1}^{\infty} 2k_1 \left\langle a_{k_1} a_{k_2} a_{k_3} \sin(\varphi_{k_1, k_2}^{k_3}) \right\rangle \end{aligned}$$

$\approx 160,000$ triads - inertial range




Triad dynamical **phase Precession**

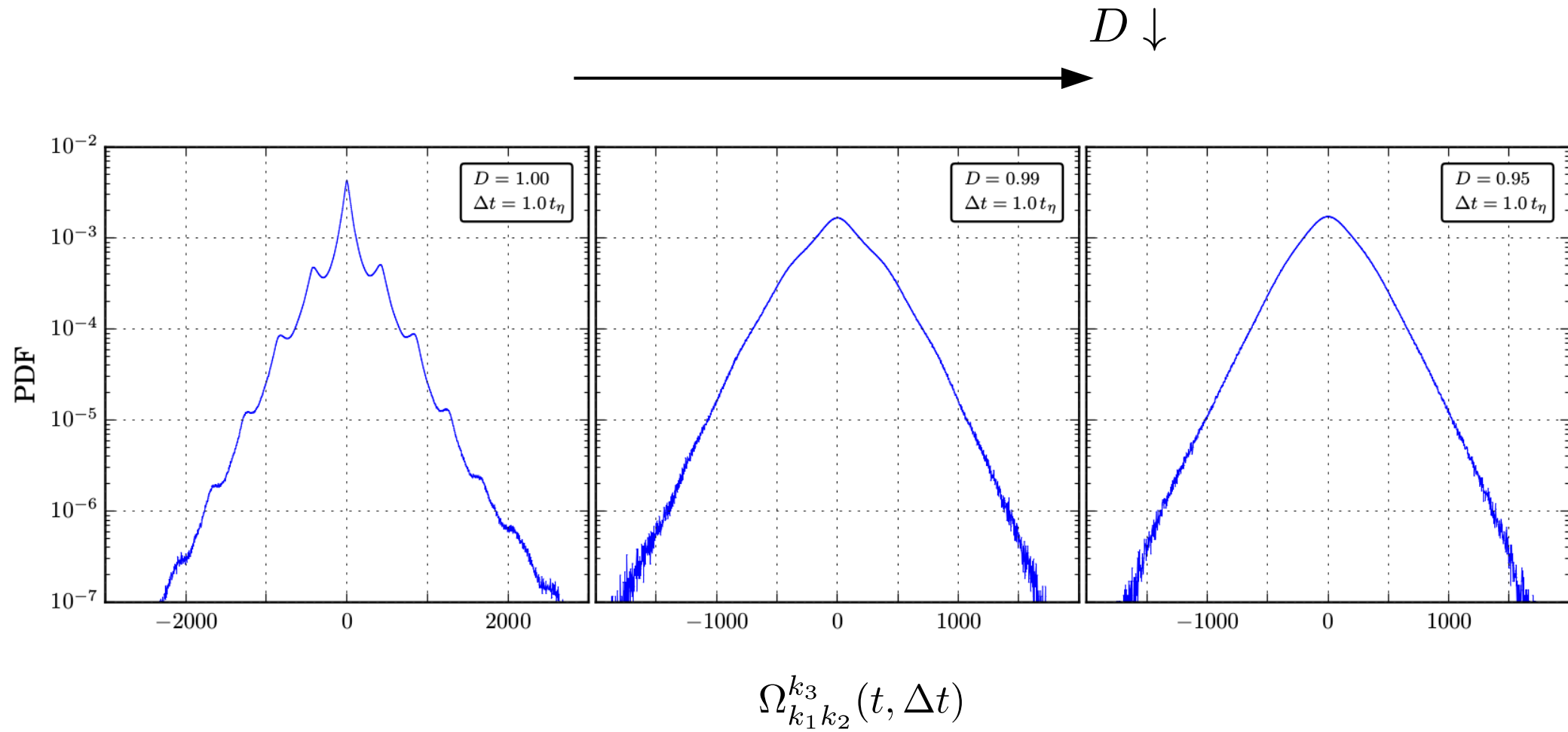
- Recall Triad dynamical Phase:

$$\varphi_{k_1 k_2}^{k_3}(t) = \phi_{k_1}(t) + \phi_{k_2}(t) - \phi_{k_3}(t)$$

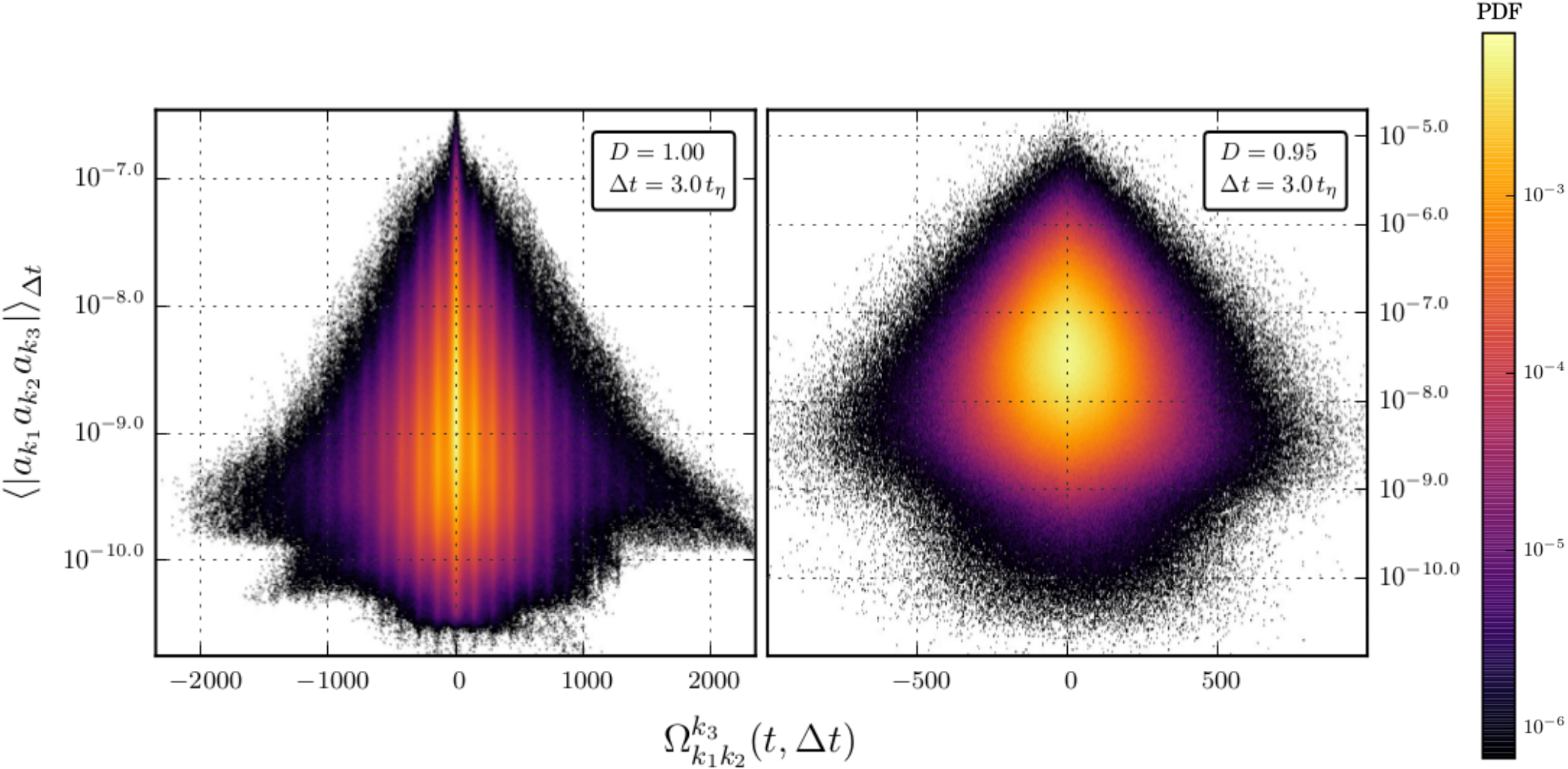
- Define the **Precession** of a triad's phase:


$$\Omega_{k_1 k_2}^{k_3} \equiv \lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t \dot{\varphi}_{k_1 k_2}^{k_3}(t') dt' \equiv \left\langle \dot{\varphi}_{k_1 k_2}^{k_3}(t') \right\rangle_{t_\infty}$$

PDF triad phase' Precession



Joint PDF, Amplitude - Precession

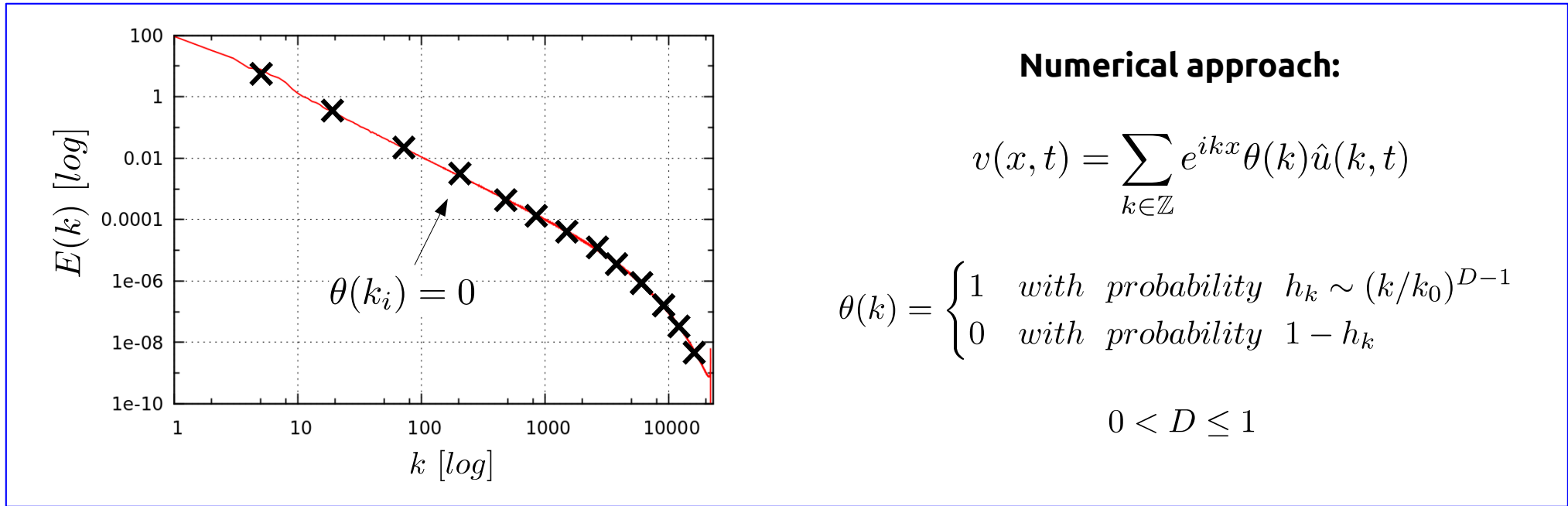


Conclusions

We studied the evolution of forced **Burgers' equation under mode reduction in Fourier space.**

- 1) Energy transfer is strongly dependent on the mode reduction protocol.
- 2) Energy focusing is the result of a global phase correlation in Fourier space.
- 3) Highlights difficulties for modelling.
- 4) Potential inspiration to search for similar phenomena in Navier-Stokes equations.

Modes Reduction, Fractal Fourier decimation:



Decimated Burgers equation:

$$\begin{aligned} \partial_t v(x, t) + P_D [v \partial_x v(x, t)] &= \\ &= \nu \partial_{xx}^2 v(x, t) + F_D \end{aligned}$$

Decimation Main Properties:

- 1) The space **dimension** can **change continuously**
- 2) The original **symmetries** of the system are kept
- 3) It acts as a **Galerkin Truncation** without the introduction of any characteristic scale
- 4) The numerical evolution can be obtained via a **psedu-spectral code**