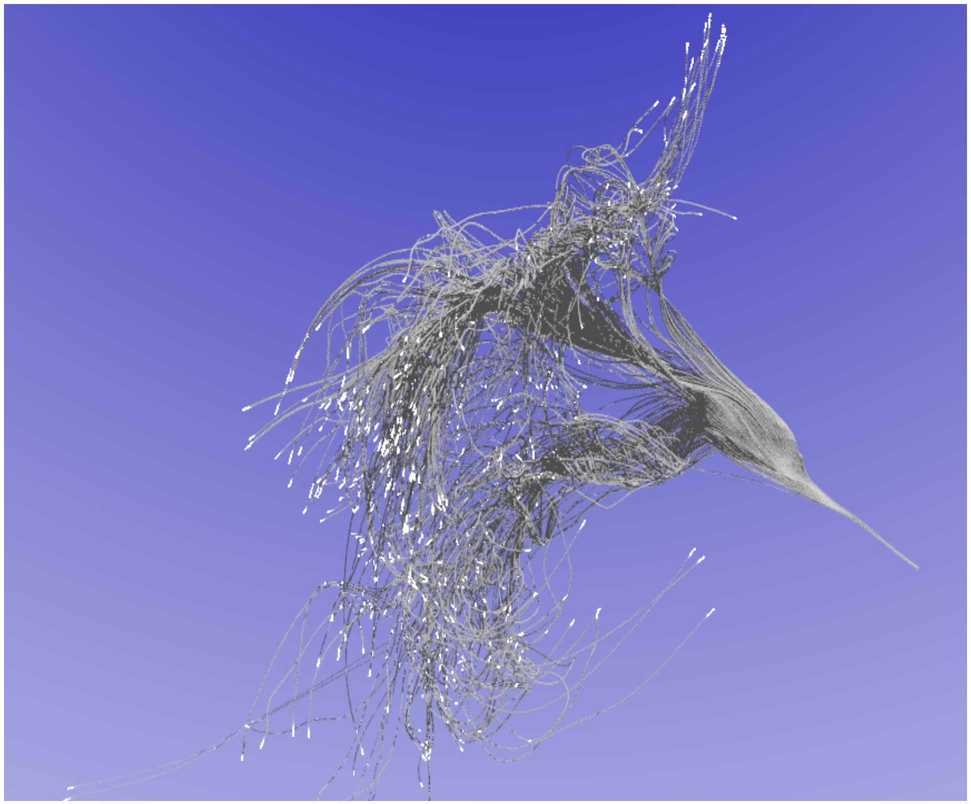
Dispersion of particles from localized sources in turbulence



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Applications

Volcano eruptions

Burning and pollutants dispersion

Plan of the talk

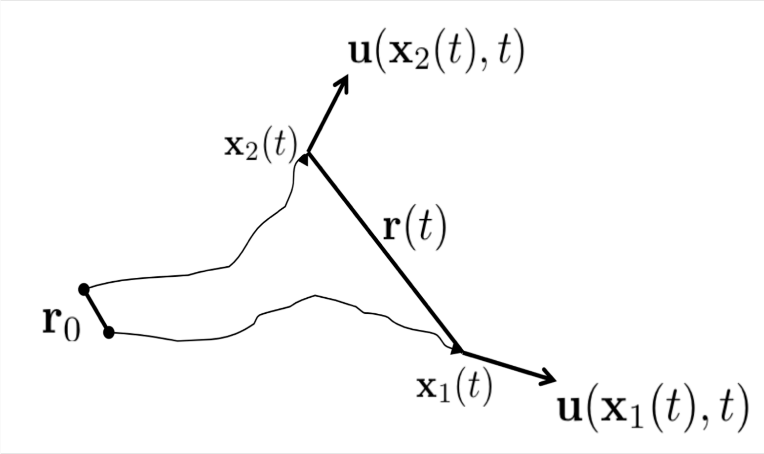
- *How pairs of tracer particles separate in homogeneous and isotropic turbulence*
- *DNS results and comparison with Richardson's PDF*
- *Model of the eddy-diffusivity to characterize the finite Reynolds number effects* on tracer particles dispersion
- *Properties of heavy particles dispersion*

Richardson's law (1926)

Diffusive process in inertial range characterized by an effective turbulent diffusivity

$$D_{Ric}(r) = \frac{1}{2}\frac{d\langle r^2\rangle}{dt} \sim \tau(r)\langle(\delta_r v)^2\rangle \sim r^{4/3}$$

Richardson's approach can be reinterpreted as the evolution of a particle pair in a stochastic Gaussian and delta-correlated in time velocity field



$$\partial_t P(r,t) = \frac{1}{r^2}\partial_r r^2 D(r)\partial_r P(r,t)$$

With $P(r,t_0) \propto \delta(r-r_0)$ the asymptotic form of *P(r, t)* is given by:

$$P(r,t) = A\frac{r^2}{(k_0\epsilon^{1/3}t)^{9/2}} exp\left[-\frac{9r^{2/3}}{4k_0\epsilon^{1/3}t}\right]$$

$$\langle r^2\rangle \simeq \epsilon t^3$$

Numerical simulation details

Fluid

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\, \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{F}$$

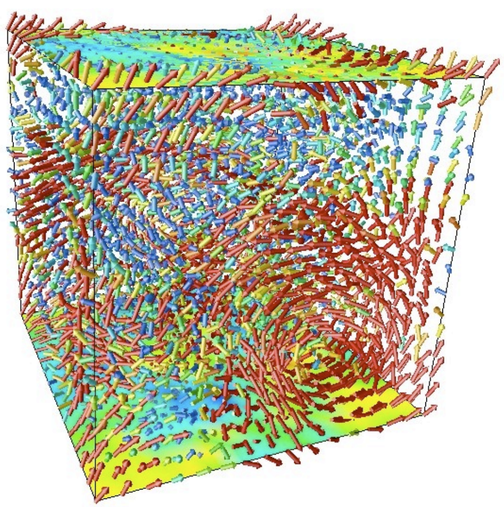
$$\nabla \cdot \mathbf{u} = 0$$

$$\dot{\mathbf{x}}_p(t) = \mathbf{u}(\mathbf{x}_p(t), t)$$ → ***Tracer particles***

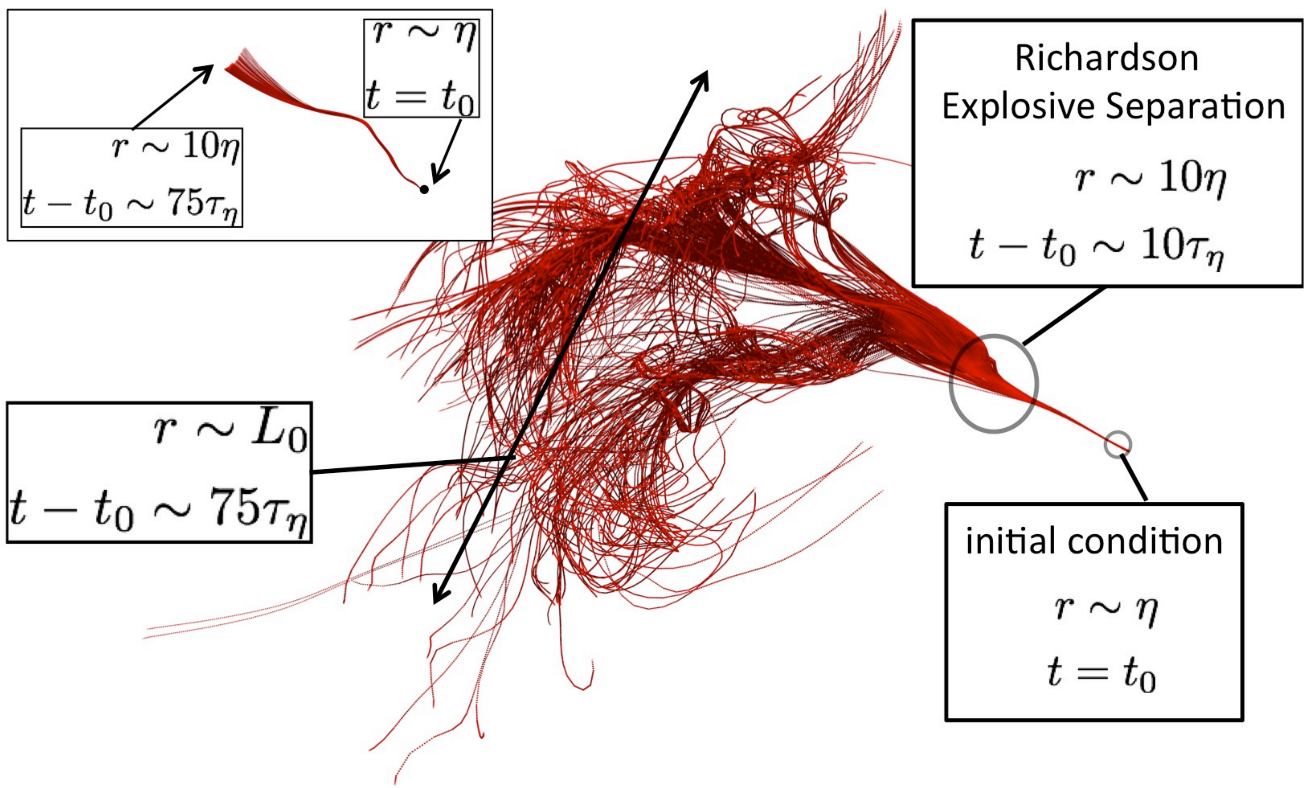
$$\frac{d\mathbf{u}_p(t)}{dt} = -\frac{1}{\tau_p}[\mathbf{u}_p(t) - \mathbf{u}(\mathbf{x}_p(t), t)]$$ → ***Heavy particles***

$$St = \tau_p / \tau_\eta$$ → ***Stokes number***

- *3-D homogeneous isotropic flow at $Re_\lambda \sim 300$*
- *Regular cubic box (1024^3 grid points) with periodic BC*
- *256 sources where anyone emits 2000 tracers and heavy particles every τ_η*
- *4 x 10^{11} particle pairs*
- *Parallel pseudo-spectral code*



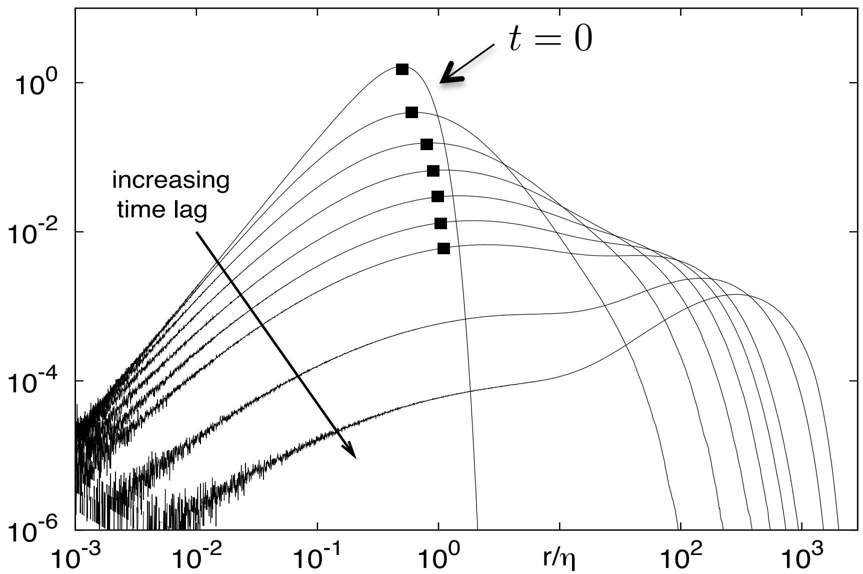
Tracers separation



We observe emissions that follow Richardson behavior and emissions which are trapped in fluid regions with a small stretching rate

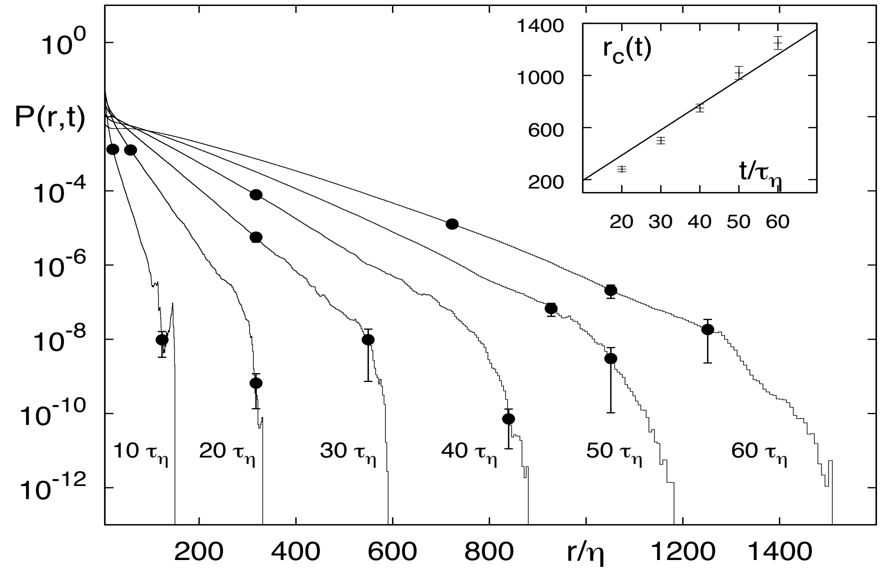
Tracers relative separation PDFs

Left PDFs tails: slowest separation events



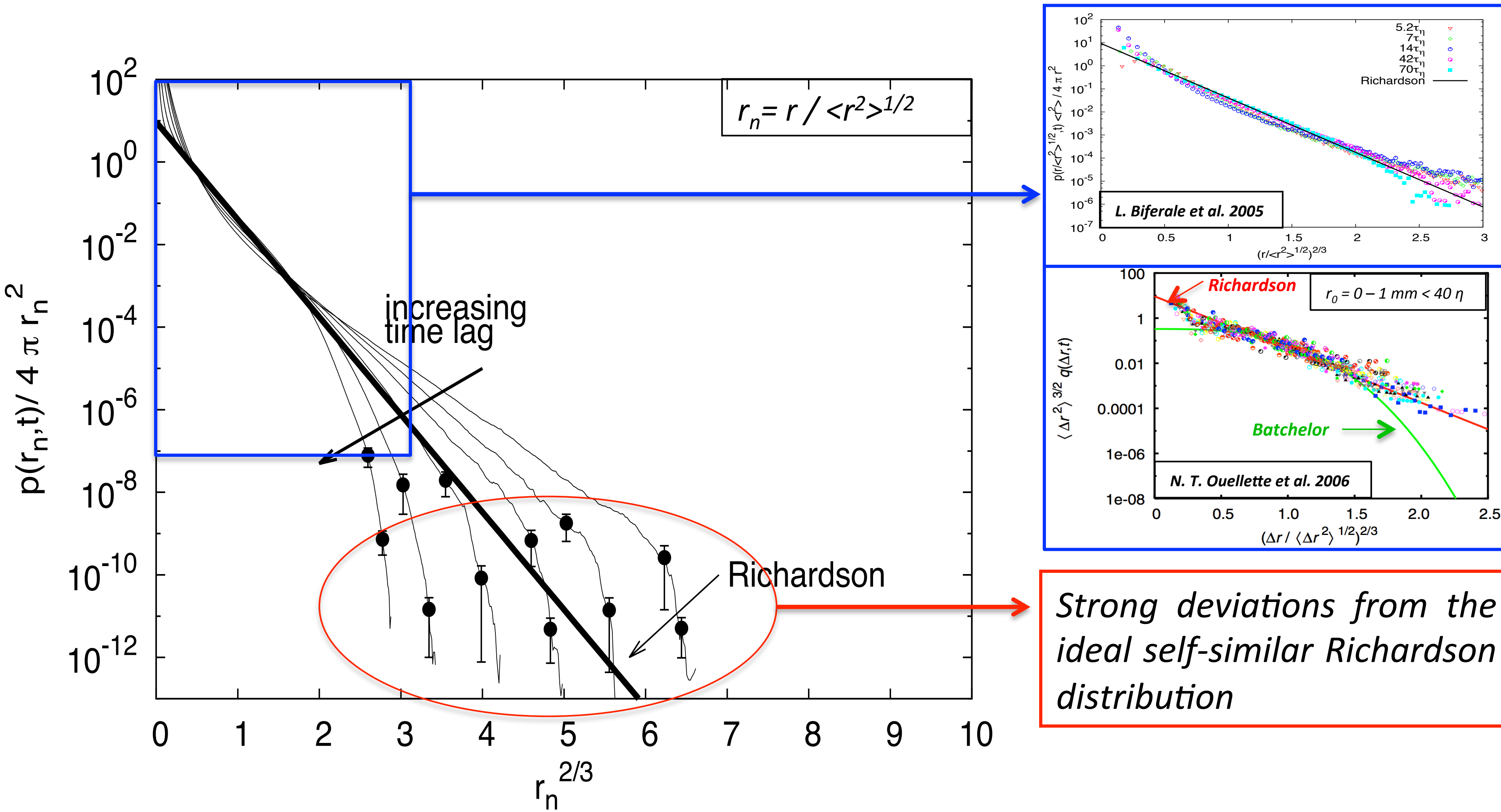
- *Presence of a peak at sub-diffusive separations at every time. This behavior is due to emissions that do not separate efficiently in the flow*
- *The PDFs at sub-diffusive separations are fitted by a Log-normal expression with a very small Lyapunov exponent*

Right PDFs tails: fastest separation events



- *Exponential-like tails with a sharp drop at a cut-off separation $r_c(t)$. This cut-off scale is the signature of tracers pairs expiring a persistent high relative velocity, which is limited by U_{rms}*
- *Supposing a linear evolution of $r_c(t)$ using U_{rms} as a travelling speed, we find a good agreement with data (see inset)*

Comparison with Richardson's PDF



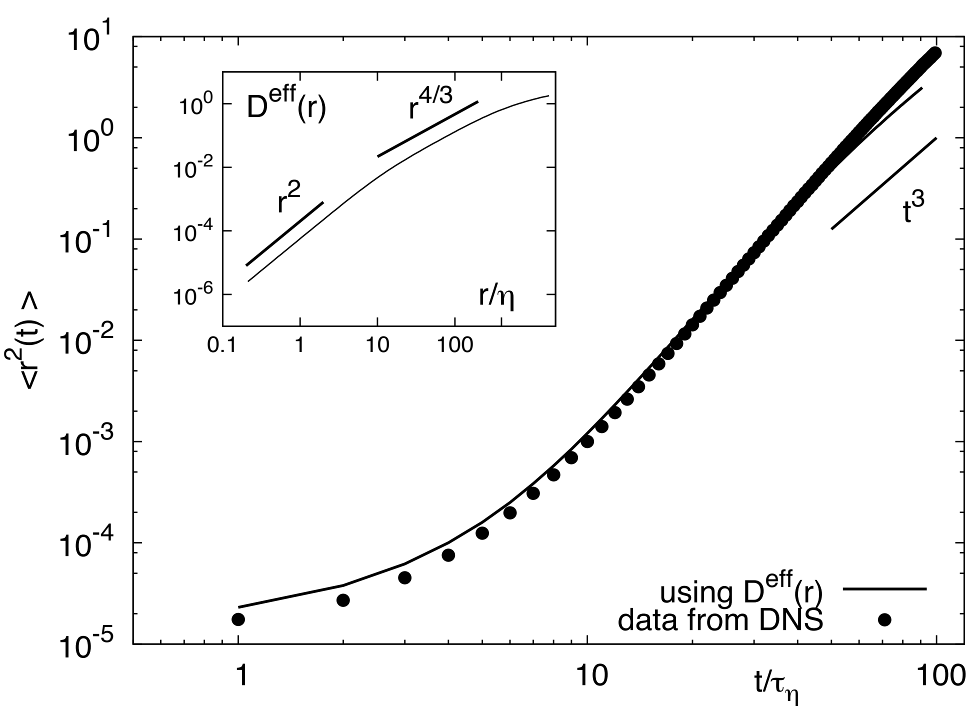
Strong deviations from the ideal self-similar Richardson distribution

These deviations can be either due to finite Reynolds effects or by neglecting temporal correlations, or both

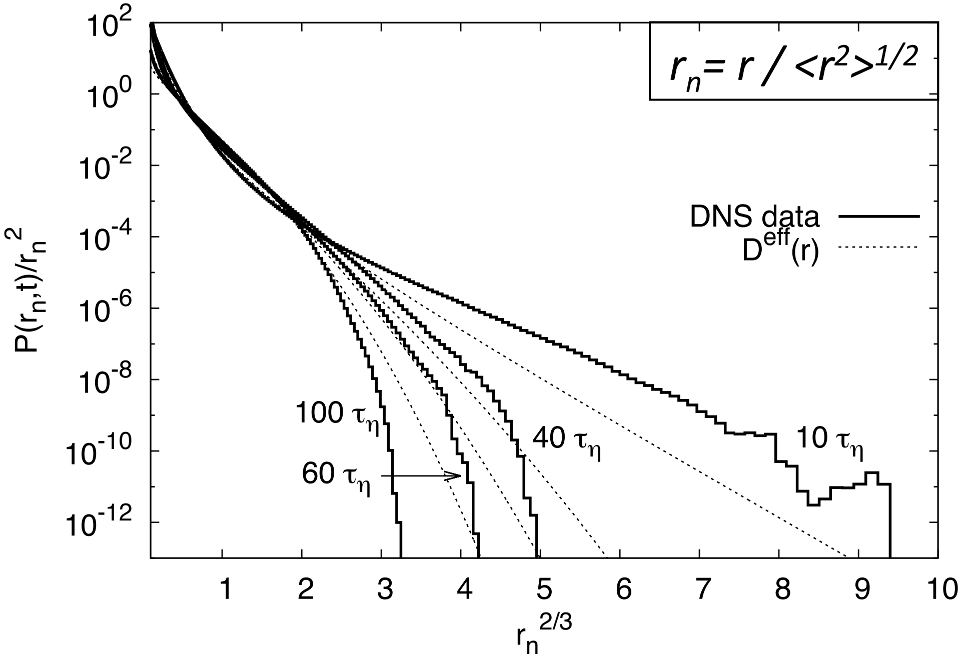
Eddy-diffusivity model (finite Reynolds effects)

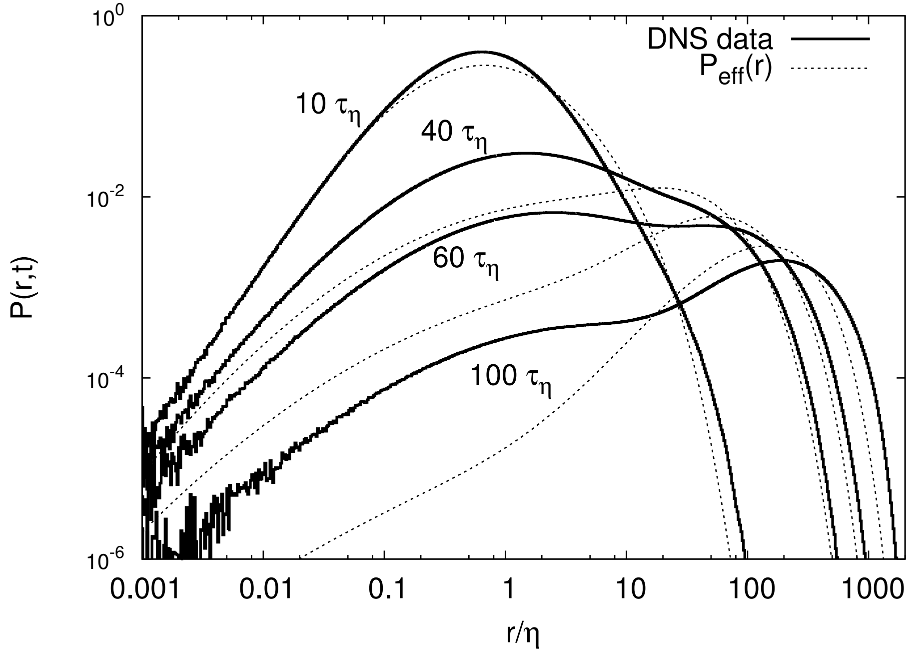
Numerical integration of Richardson diffusive equation using an effective turbulent eddy-diffusivity that keeps in to account the viscous and large scale cut-offs

$$D_{||}^{eff}(r) \sim \tau(r)\langle(\delta_r v)^2\rangle \Longrightarrow \begin{cases} D_{||}^{eff}(r) \sim r^2 & r \ll \eta \\ D_{||}^{eff}(r) \sim r^{4/3} & \eta \ll r \ll L_0 \\ D_{||}^{eff}(r) \sim const. & r \gg L_0 \end{cases}$$



Model-DNS compared

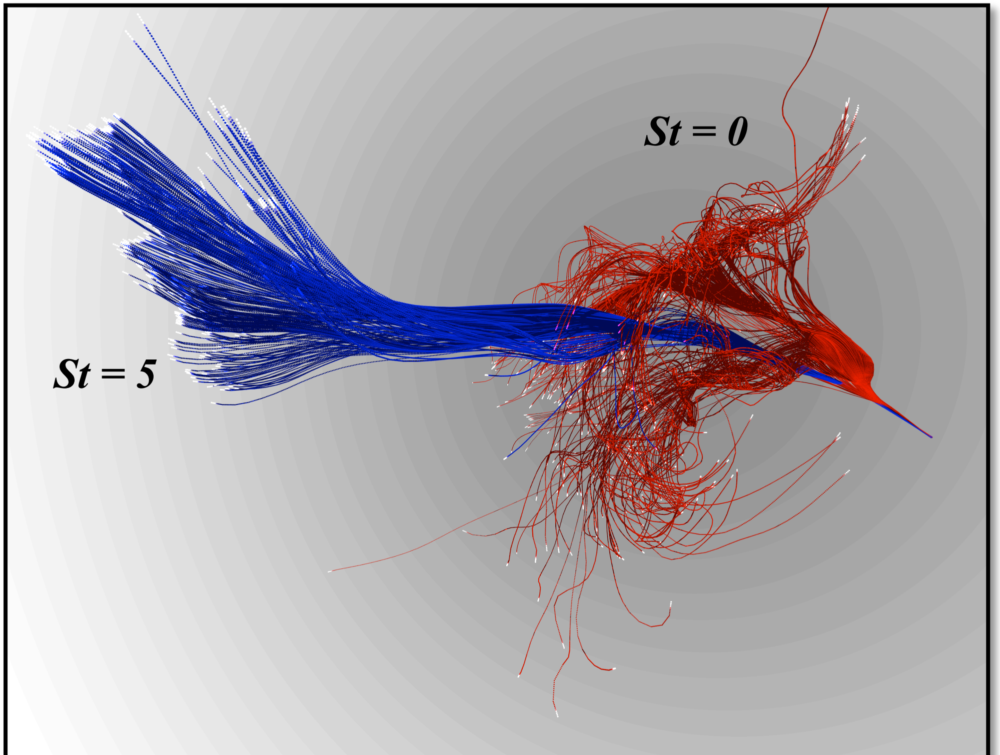




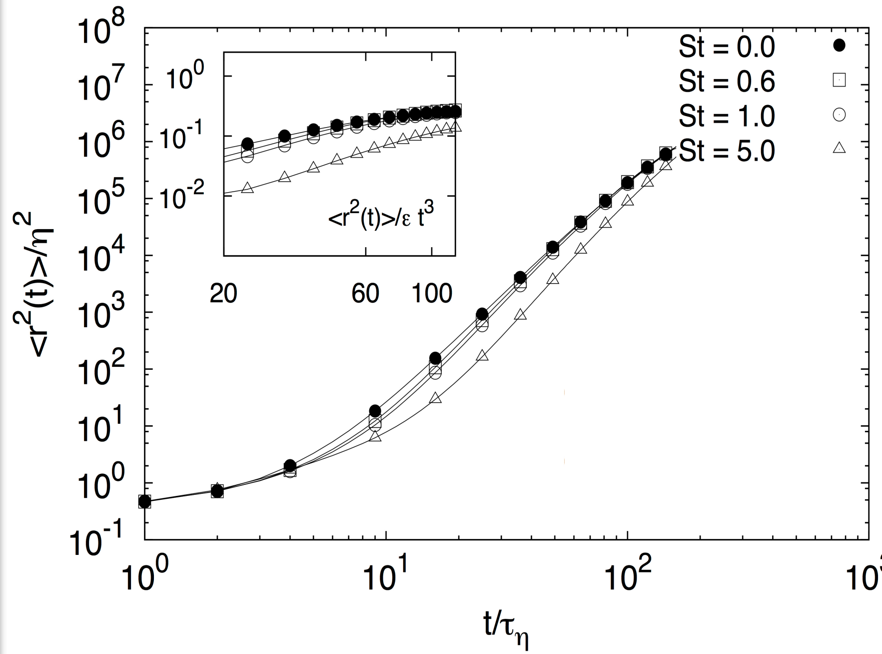
The model breaks the ideal self-similar PDF behavior but it is not able to reproduce the sharp drop at large scales

The model is not able to describe the peak at diffusive scales

Inertial heavy particles dispersion

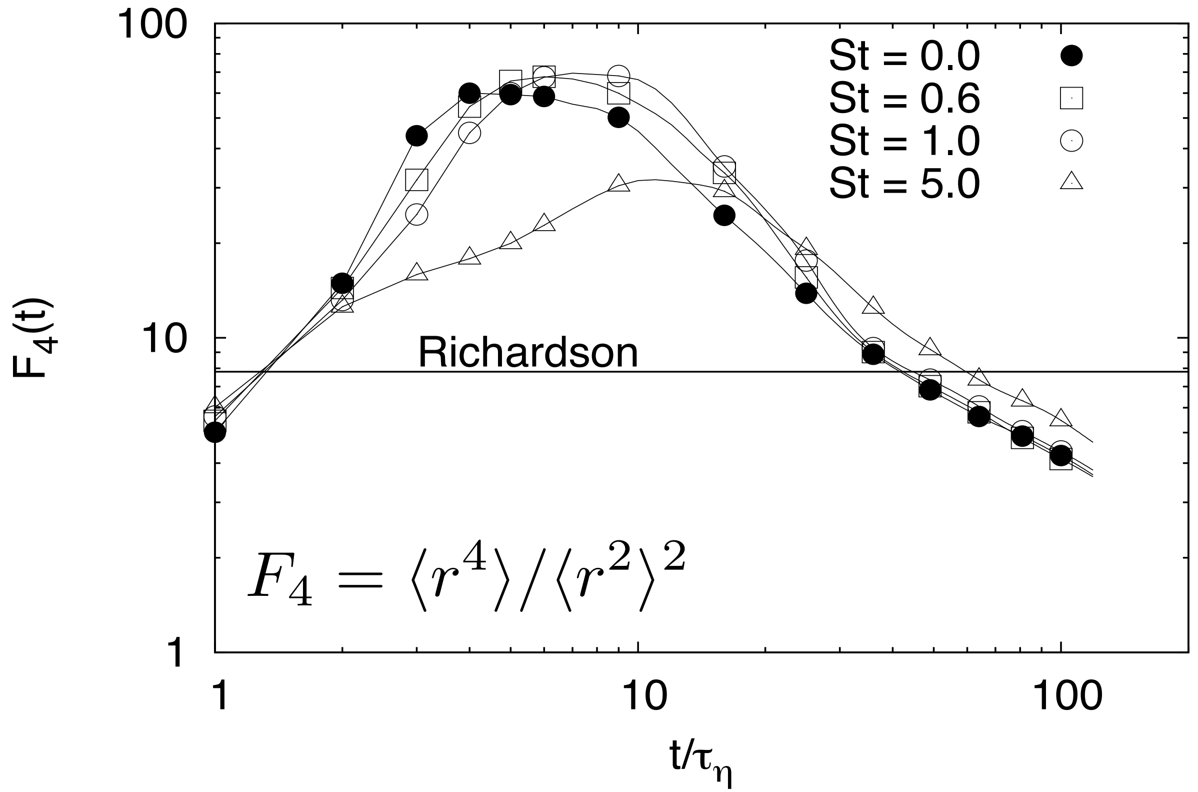


Evolution of tracers (red) and heavy particle (blue) up to 75 τ_η emitted from the same source at the same time



The heavy particles accumulate a temporal delay which is recovered at large time

Viscous filtering effect of heavy particles



Heavy particles with St = 5 filter-out the viscous fluctuations of the underlying fluid

Conclusion

- *Thanks to our huge statistics we are able to quantify the deviations from Richardson behavior of tracers particles dispersion.*

- *By using a model that keeps into account viscous inertial and large scale physics, we got a good agreement with DNS data.*

- *Due to the inertia, heavy particles filter-out the viscous fluid fluctuations.*