Copenhagen September 14th-18th 2014 EFMC10

Deformation statistics of sub-Kolmogorov-scale ellipsoidal drops in isotropic turbulence

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MOTIVATION

Droplet dynamics in turbulence is a key issue for many engineer... se"

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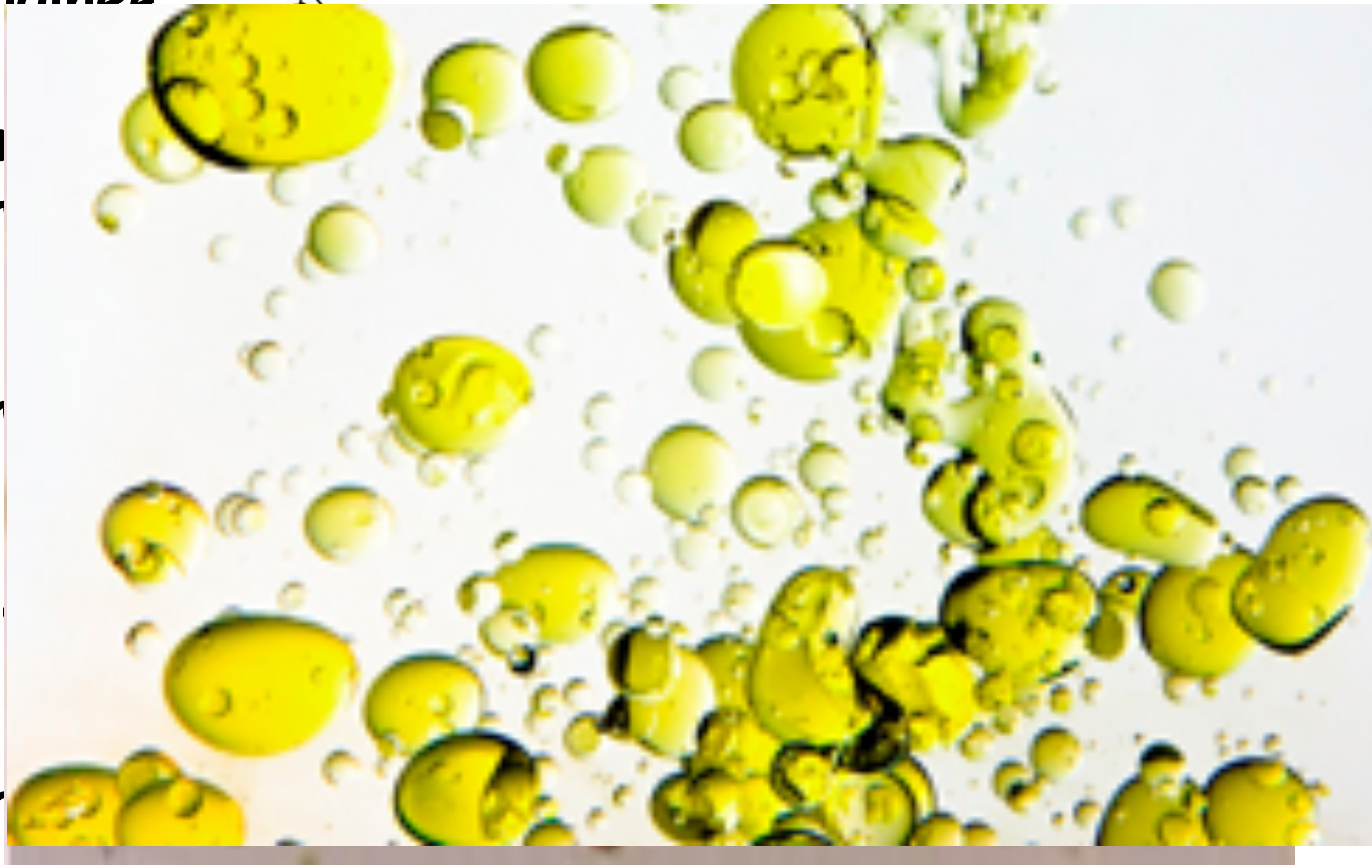

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BACKGROUND

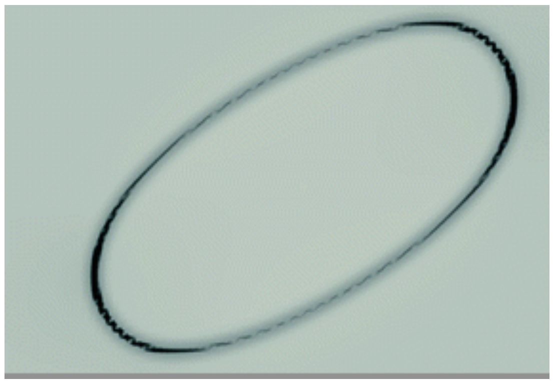
The sub-Kolmogorov size of the droplets implies that:

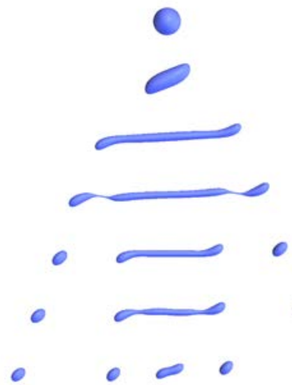
only viscous drag induced by the shear can distort the droplet shape (no inertial forces)

the distortion is resisted by the surface tension that tends to restore the spherical shape

Initial study by Taylor (1932) with drops in a laminar flow

Frijters et al. (2012)





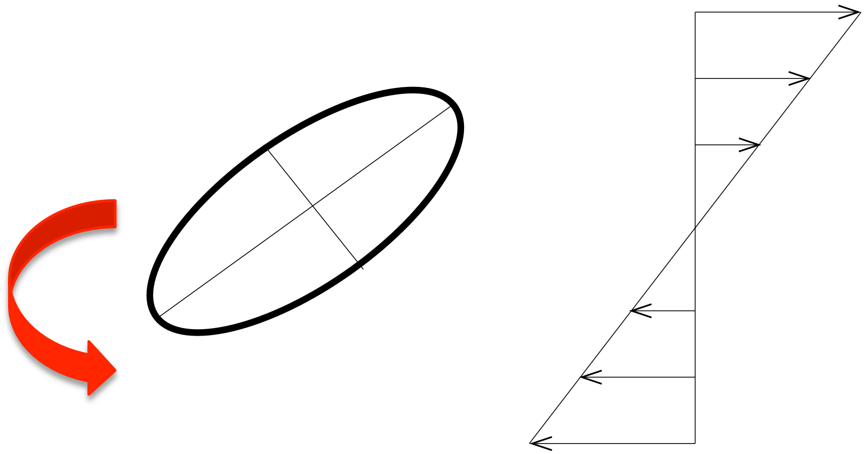
Komrakova et al. (2012)

Equilibrium configuration or droplet break-up

See Kolmogorov (1949) and Hinze (1955) or Lasheras et al. (2002) for larger drops with inertia forces

BACKGROUND

In laminar flows the shear can be characterized just by one (or few) parameters



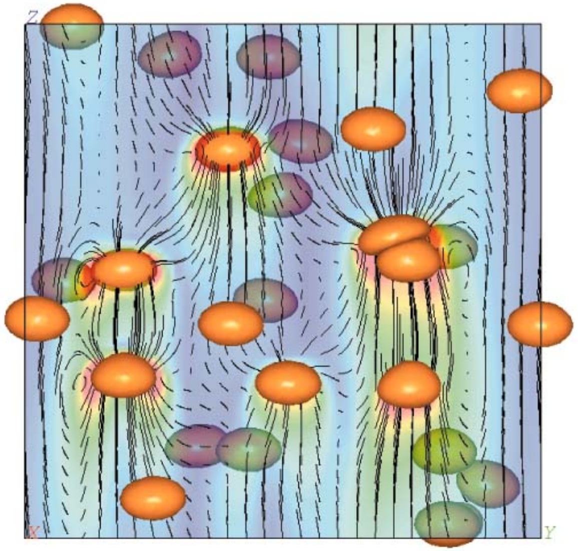
In contrast, in turbulent flows there is a wide range of shears that locally (in space and time) can exceed the average by orders of magnitude.

Accordingly a fraction of the droplets can undergo break-up while others will survive → statistical characterization

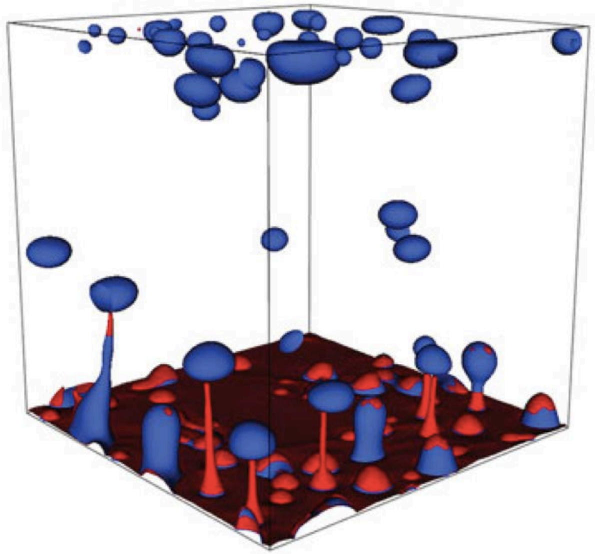
The vorticity decorrelates the strain rate with the droplet deformation ……

BACKGROUND

Detailed simulations resort to DNS of turbulence coupled with boundary integral methods (or similar) [Cristini et al. (2003), Terashima & Tryggvason (2009) and Can & Prosperetti (2012)]



http://www3.nd.edu/~gtryggva/MCFD/



Biferale et al. PRL (2012)

☺ Highly complex droplets shapes, instabilities, necks and satellite droplets are captured

☹ Turbulence Re_λ only moderately high and the number of droplets is few tens or hundreds

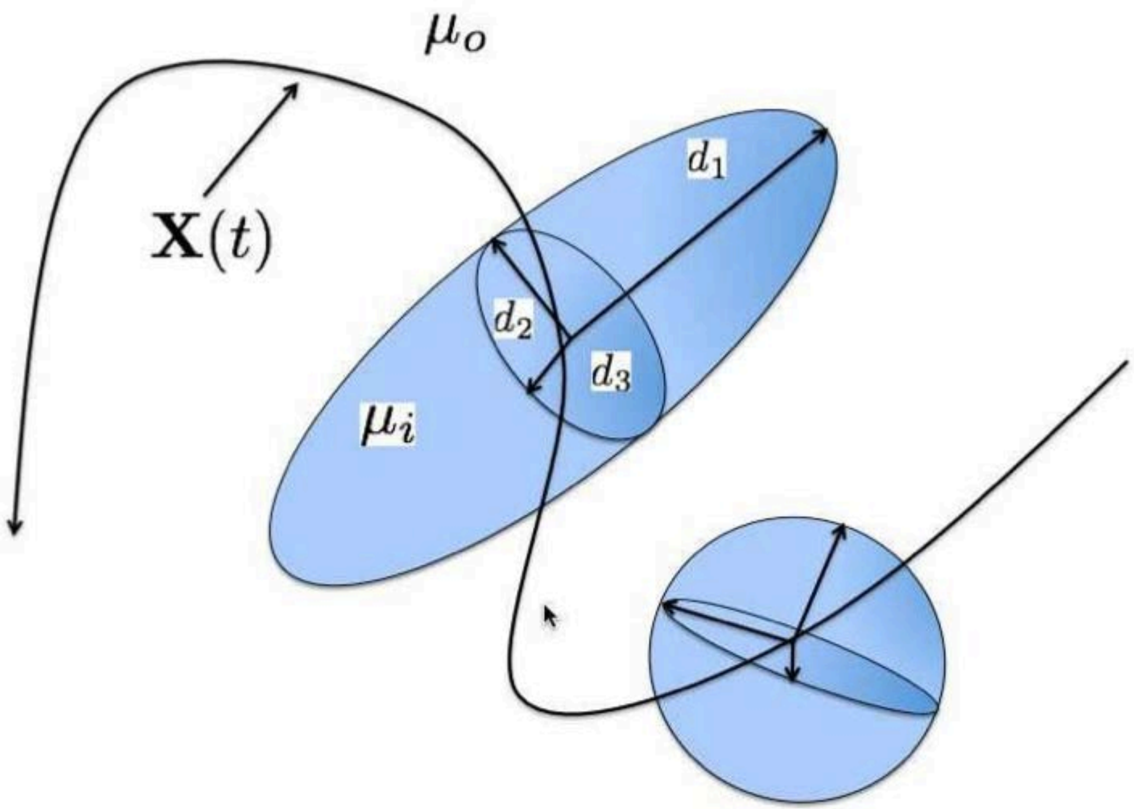
The Model

Pointwise Lagrangians with some physics built around

A fluid of viscosity μ_0 in turbulent motion

Droplets of an immiscible fluid of viscosity μ_1

Surface tension Λ at the interface



Initially spherical droplets can deform only to (triaxial) ellipsoids

The Model

A second-order symmetric positive-definite tensor **M** is evolved in time

Its eigenvalues and eigenvectors yield the squared semi-axes of the ellipsoid and their orientation

Maffettone & Minale (1998)

$$\frac{dM_{ij}}{dt} = \Omega_{ik}M_{kj} - M_{ik}\Omega_{kj} + f_2(\mu)(S_{ik}M_{kj} + M_{ik}S_{kj}) - \frac{f_1(\mu)}{\tau}(M_{ij} - g(\mathrm{II}_M, \mathrm{III}_M)\delta_{ij})$$

$$\mu = \mu_i/\mu_o \qquad \tau = \mu_o R/\Lambda \qquad g(\mathrm{II}_M, \mathrm{III}_M) = 3\frac{\mathrm{III}_M}{\mathrm{II}_M}$$

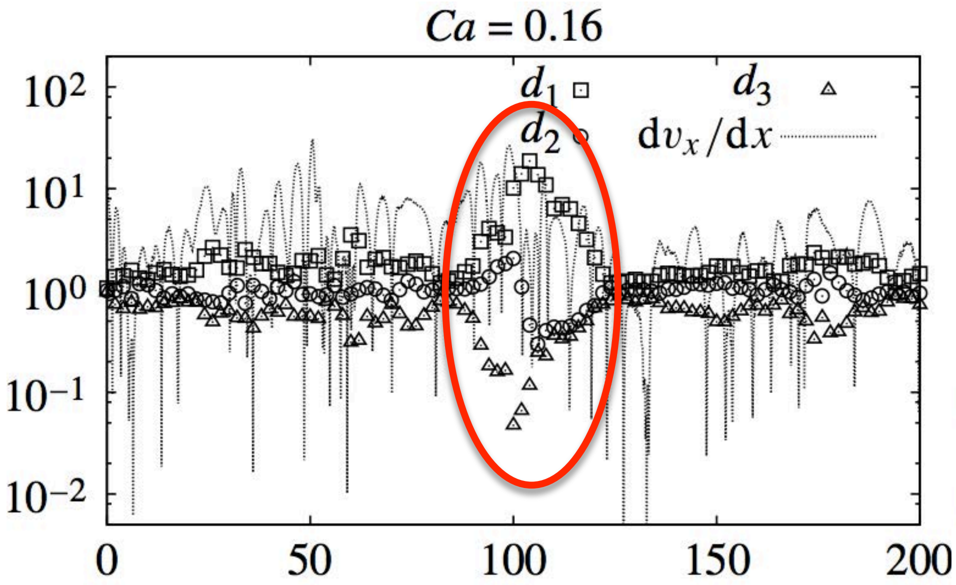
$$\Omega_{kj} = 0.5(\partial_j u_k - \partial_k u_j) \qquad S_{kj} = 0.5(\partial_j u_k + \partial_k u_j)$$

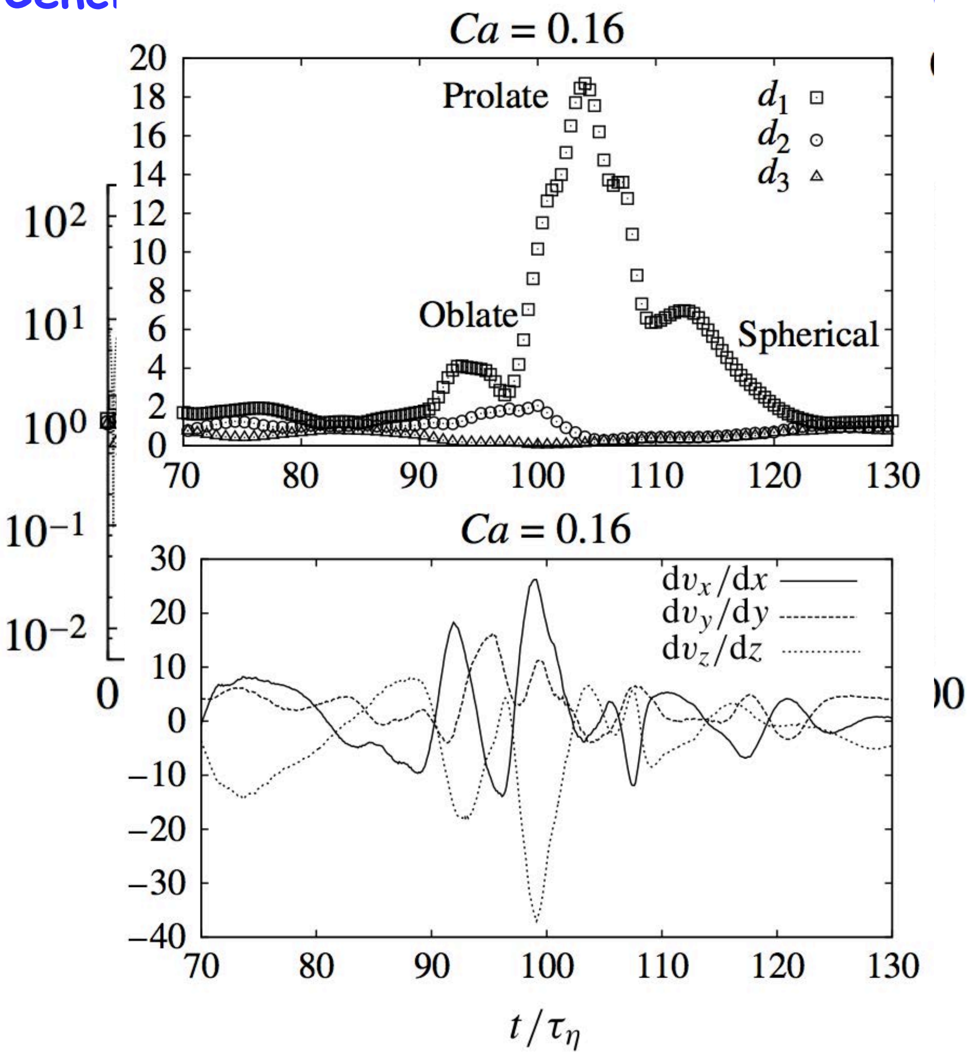
$$f_1(\mu) = \frac{40(\mu+1)}{(2\mu+3)(19\mu+16)} \qquad f_2(\mu) = \frac{5}{2\mu+3}$$

Results μ=1

M is advanced in time along Lagrangian trajectories computed from forced homogeneous isotropic turbulence at Re_λ=185 (512^3) and Re_λ=400 (2048^3)

Cencini et al (2006), Bec et al. (2010)

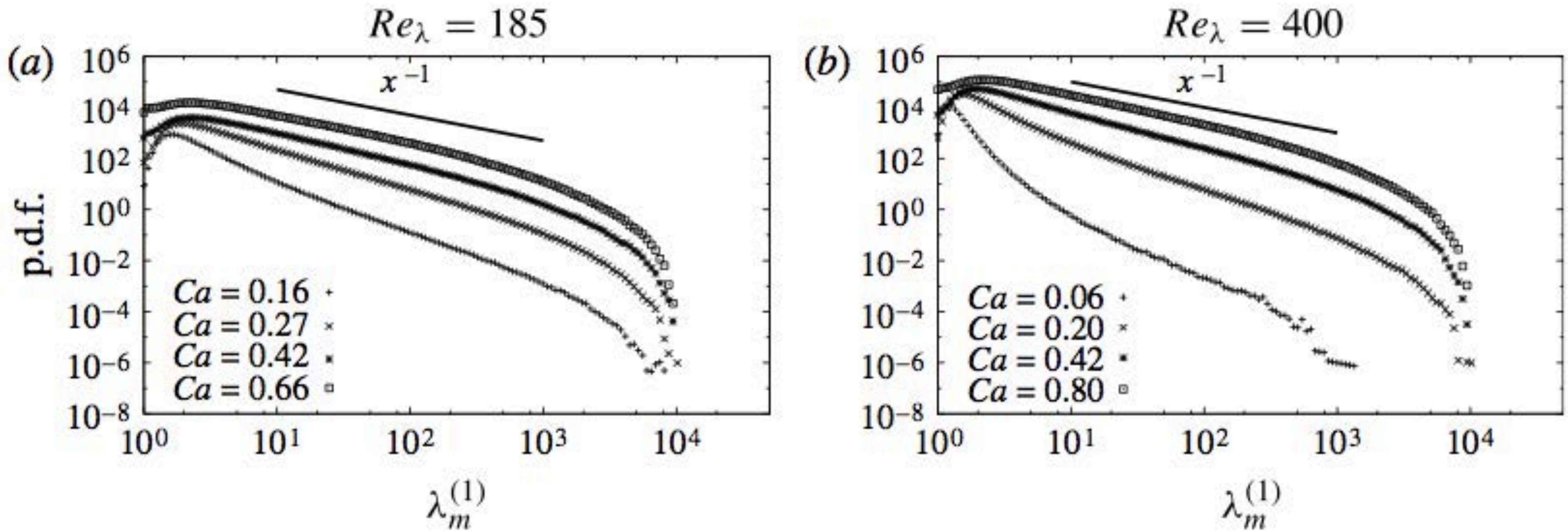




1.5×10^4 trajectories at Re_λ=185

Results

A droplet undergoes break-up when the ratio between the largest to the smallest semi-axis exceeds a given threshold $d_1/d_3 \geq 10^3$ (arbitrary value but the slopes do not depend on the threshold)



The p.d.f.s saturate to a slope -1 for $Ca \geq 0.4$ that we interpret as a critical Capillary number for all the droplets to break up

Theoretical explanation

For $\mu=1$ ➔ $f_2(\mu)=1$ and the Maffettone & Minale (1998) model becomes

$$\frac{dM_{ij}}{dt} = (A_{ik}M_{kj} + M_{ik}A_{kj}) - \frac{f_1}{\tau}\left(M_{ij} - g(\mathrm{III}_M, \mathrm{II}_M)\delta_{ij}\right) \quad \text{with} \quad A_{ik} = \partial u_i / \partial x_k$$

In the tail of the p.d.f ($\mathrm{Tr}(\boldsymbol{M}) \gg 1$) it is $M_{ij} - g\delta_{ij} \approx M_{ij}$
the largest eigenvalue dominates M and $M_{ij} \sim R_i R_j$
(R is the end-to-end vector of a polymer)

The first part is just the deformation of an infinitesimal fluid volume by the velocity gradient ➔ Liapunov statistics applies

As in Balkowsky et al. (2000) or Boffetta et al. (2003) we focus in Tr(M) whose p.d.f. slope will be twice that of R

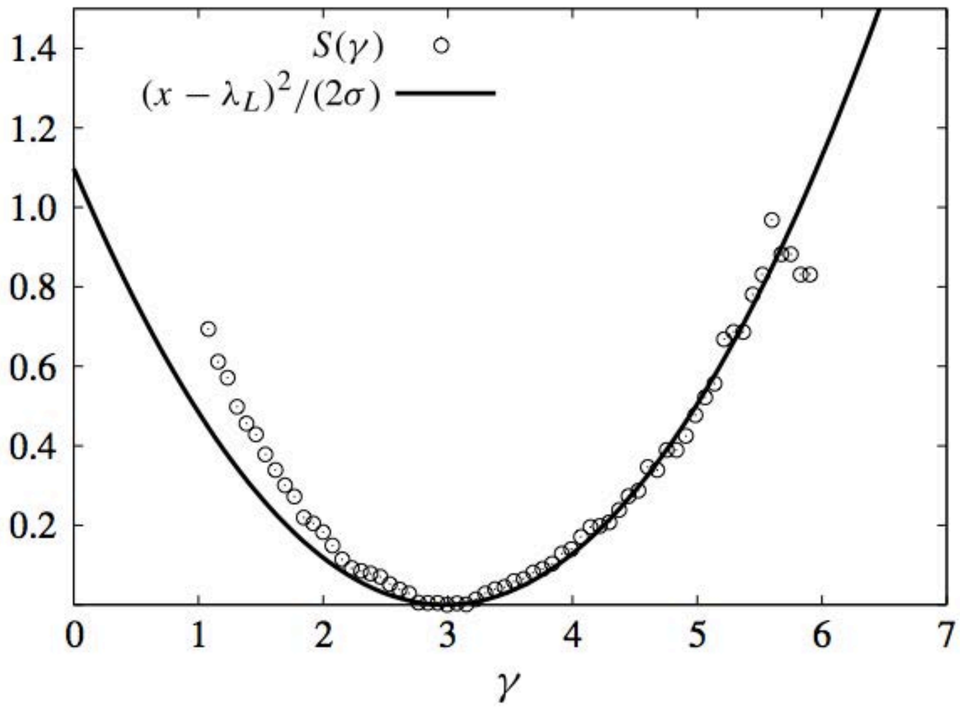
Theoretical explanation

The statistics of R evolves following the finite-time-Liapunov exponent (FTLE) Boffetta et al. (2003)

$$\gamma(t) = \frac{1}{t} \log \left(\frac{|\boldsymbol{R}(t)|}{|\boldsymbol{R}(0)|} \right)$$

and the p.d.f. is given by (Frisch 1995) $P(\gamma, t) \sim \exp(-tS(\gamma))$

With S(Υ) the Cramer function



from Bec et al. (2006)

Theoretical explanation

… after some arithmetics

$$\langle [\mathrm{Tr}(\boldsymbol{M}(t))]^q \rangle \sim \exp\left[t \left(L(2q) - q\frac{f_1}{\tau} \right) \right]$$

that, in order to exist a stationary p.d.f., must be

$$\lim_{q \to 0} [L(2q) - q f_1 / \tau^c] = 0$$

(normalizable at all times),

This yields a prediction:

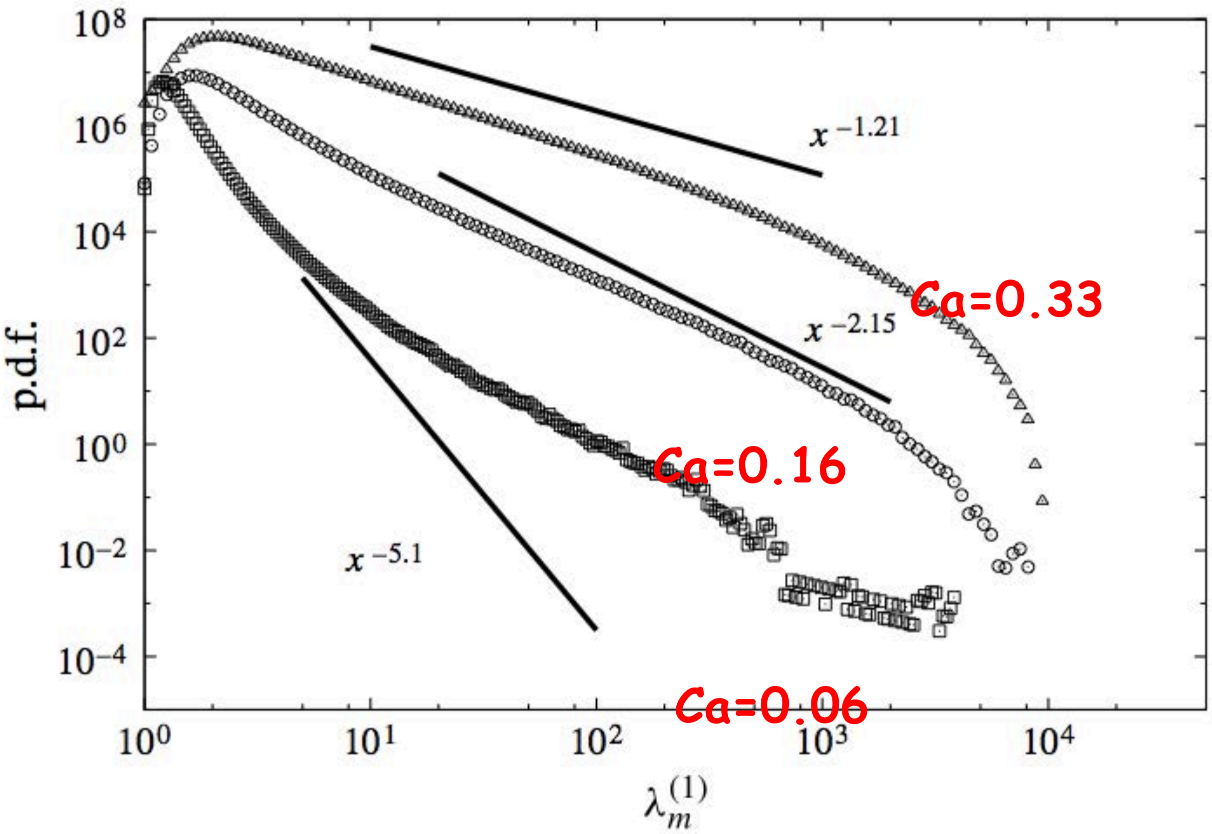
for the critical relaxation time $\tau^c = f1/(2\lambda_L)$

and for the slope of the p.d.f. tail $-[1+q(\tau)]$

($q(\tau)$ is the largest order of non diverging moment)

Theoretical explanation

… using the DNS data at Re_λ=185 (f_1=0.457, Λ_L=2.97) it is obtained τ^c=0.077 which yields Ca_c = 0.42



The slope prediction get increasingly better as Ca →Ca_c.
For Ca<< Ca_c the stretching is unimportant and the FTLE might not apply

CLOSING REMARKS

Similar predictions (and agreement at $Re_\lambda=400$)

For $\mu\neq1$ (different viscosities) the importance of strain and rotation is unequal and the analogy with polymers fails (the prediction underestimates Ca_c)

For further statistics on alignments and deformations and more analytical details see

J. Fluid Mech. (2014), *vol.* 754, *pp.* 184–207. © Cambridge University Press 2014 184
doi:10.1017/jfm.2014.366

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