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UNIVERSITÀ DEGLI STUDI DI ROMA

Inertial transfer and small-scale structures in magnetohydrodynamic turbulence

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DFG Turbulent Superstructures

MHD equations

$$\begin{cases} \partial_t u_i + \partial_j \left(u_i u_j\right) - \partial_j \left(b_i b_j\right) = -\partial_j p \, \delta_{ij} + \nu \, u_{i,jj} + f_i \\ \partial_t b_i + \partial_j \left(b_i u_j - u_i b_j\right) = \eta \, b_{i,jj} \end{cases} \qquad p = p^* + \frac{b^2}{2}$$

Coarse-grained fields → Gaussian filter

$$\overline{u}_i^{\ell}(\boldsymbol{x}) = \int_\Omega d\boldsymbol{r} \; G^{\ell}(\boldsymbol{r}) u_i(\boldsymbol{x}+\boldsymbol{r}).$$

$$\overline{b}_i^{\ell}(\boldsymbol{x}) = \int_\Omega d\boldsymbol{r} \; G^{\ell}(\boldsymbol{r}) b_i(\boldsymbol{x}+\boldsymbol{r}).$$

$$G^{\ell}(\boldsymbol{r}) = \frac{1}{(2\pi\ell^2)^{3/2}} \exp\left(-\frac{|\boldsymbol{r}|^2}{2\ell^2}\right)$$

$$\begin{cases} \partial_t \overline{u}_i^\ell + \partial_j \left(\overline{u}_i^\ell \overline{u}_j^\ell - \overline{b}_i^\ell \overline{b}_j^\ell - \tau_{ij}^{I,\ell} + \tau_{ij}^{M,\ell} + \overline{p}^\ell \delta_{ij} - 2\nu \overline{S}_{ij}^\ell \right) = \overline{f}_i^\ell \\ \partial_t \overline{b}_i^\ell + \partial_j \left(\overline{b}_i^\ell \overline{u}_j^\ell - \overline{u}_i^\ell \overline{b}_j^\ell - \tau_{ij}^{D,\ell} + \tau_{ij}^{A,\ell} - 2\eta \overline{\Sigma}_{ij}^\ell \right) = 0 \end{cases}$$

$$\tau_{ij}^{I,\ell} = \overline{u_i u_j}^\ell - \overline{u}_i^\ell \overline{u}_j^\ell$$ Inertial SGS tensor

$$\tau_{ij}^{M,\ell} = \overline{b_i b_j}^\ell - \overline{b}_i^\ell \overline{b}_j^\ell$$ Maxwell SGS tensor

$$\tau_{ij}^{D,\ell} = \overline{u_i b_j}^\ell - \overline{u}_i^\ell \overline{b}_j^\ell$$ Dynamo SGS tensor

$$\tau_{ij}^{A,\ell} = \overline{b_i u_j}^\ell - \overline{b}_i^\ell \overline{u}_j^\ell$$ Advection SGS tensor

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

$$\Sigma_{ij} = \frac{1}{2}\left(\frac{\partial b_i}{\partial x_j} + \frac{\partial b_j}{\partial x_i} \right)$$

Offermans, G. P., Biferale, L., Buzzicotti, M., & Linkmann, M. POP (2018)

Aluie H. NJP (2017)

$$\partial_t \frac{1}{2}\left(\overline{u_i}^\ell \overline{u_i}^\ell + \overline{b_i}^\ell \overline{b_i}^\ell\right) + \partial_j(\ldots) = -\Pi^{I,\ell} + \Pi^{M,\ell} - \Pi^{A,\ell} + \Pi^{D,\ell} + \overline{u_i}^\ell \overline{f_i}^\ell - 2\nu \overline{S_{ij}}^\ell \overline{S_{ij}}^\ell - 2\eta \overline{\Sigma_{ij}}^\ell \overline{\Sigma_{ij}}^\ell$$

boundary term (vanishes in mean)

energy fluxes across the scales

input energy from forcing

viscous dissipation

$$\Pi^{I,\ell} = -\overline{S}^\ell_{ij} \tau^{I,\ell}_{ij}$$ Inertial flux

$$\Pi^{M,\ell} = -\overline{\Sigma}^\ell_{ij} \tau^{M,\ell}_{ij}$$ Maxwell flux

$$\Pi^{D,\ell} = -\overline{b}^\ell_{i,j} \tau^{D,\ell}_{ij}$$ Dynamo flux

$$\Pi^{A,\ell} = -\overline{b}^\ell_{i,j} \tau^{A,\ell}_{ij}$$ Advection flux

MHD energy fluxes

$$\Pi^\ell = \Pi^{I,\ell} - \Pi^{M,\ell} + \Pi^{A,\ell} - \Pi^{D,\ell}$$

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$$\partial_t \frac{1}{2}\left(\overline{u_i}^{\ell}\overline{u_i}^{\ell} + \overline{b_i}^{\ell}\overline{b_i}^{\ell}\right) + \partial_j(\ldots) = -\Pi^{I,\ell} + \Pi^{M,\ell} - \Pi^{A,\ell} + \Pi^{D,\ell} + \overline{u_i}^{\ell}\overline{f_i}^{\ell} - 2\nu\overline{S_{ij}}^{\ell}\overline{S_{ij}}^{\ell} - 2\eta\,\overline{\Sigma_{ij}}^{\ell}\,\overline{\Sigma_{ij}}^{\ell}$$

→ $\Pi^{I,\ell} = -\overline{S}^{\ell}_{ij}\tau^{I,\ell}_{ij}$ Inertial flux

→ $\Pi^{M,\ell} = -\overline{\Sigma}^{\ell}_{ij}\tau^{M,\ell}_{ij}$

→ $\Pi^{D,\ell} = -\overline{b}^{\ell}_{i,j}\tau^{D,\ell}_{ij}$

→ $\Pi^{A,\ell} = -\overline{b}^{\ell}_{i,j}\tau^{A,\ell}_{ij}$

MHD energy fluxes

$$\Pi^{\ell} = \boxed{\Pi^{I,\ell}} - \Pi^{M,\ell} + \Pi^{A,\ell} - \Pi^{D,\ell}$$

2 DNS datasets: HD, MHD

- Mhd: → nonlinear saturated dynamo $Pm = 1$
- Hydro: → homogeneous and isotropic turbulence
- statistically stationary
- Forcing $\boldsymbol{f}$: random process active in the large-scales
  - → hydro: gaussian process, $k_{\mathrm{f}} \in [0.5, 2.4]$
  - → mhd: Ornstein-Uhlenbeck $k_{\mathrm{f}} \in [3, 5]$ w/ minimal cross-helicity injection
- periodic BC on domain $[0, 2\pi]^3$ → pseudo-spectral method
- 1024^3 collocation points
  - → hydro: $k_{\max}\eta = 1.43$
  - → mhd: $k_{\max}\eta_u = 1.46$
    $k_{\max}\eta_b = 1.17$

*for hydro see run 22 of Sahoo, G., Bonaccorso, F., Biferale L. (2015)

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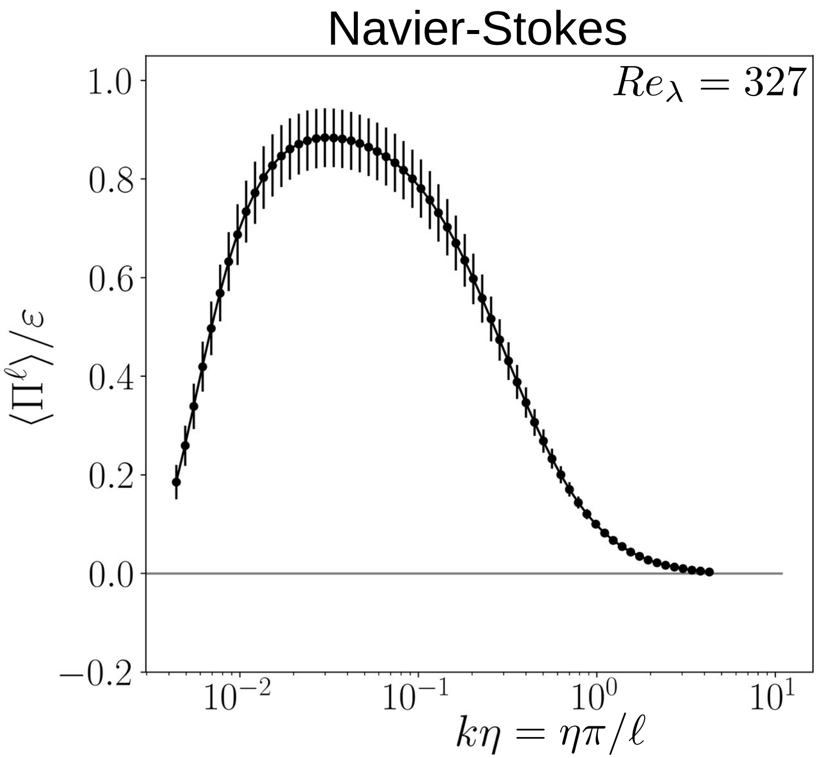
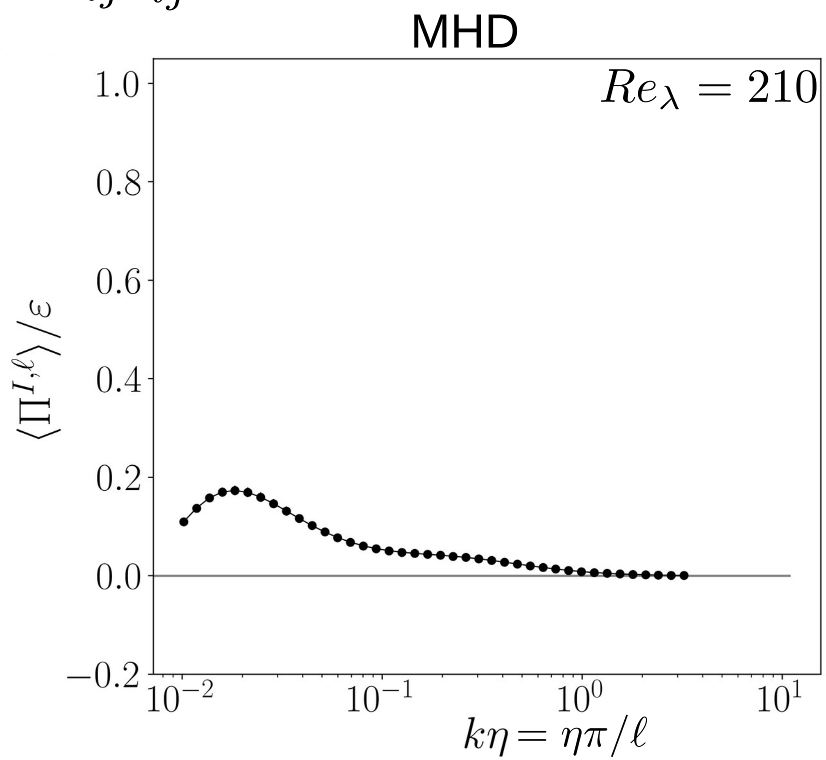
HD versus **MHD**: inertial flux $\boldsymbol{u} \cdot \nabla \boldsymbol{u}$

$$\Pi^{I,\ell} = -\overline{S}_{ij}^{\ell} \tau_{ij}^{I,\ell}$$





$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}$$

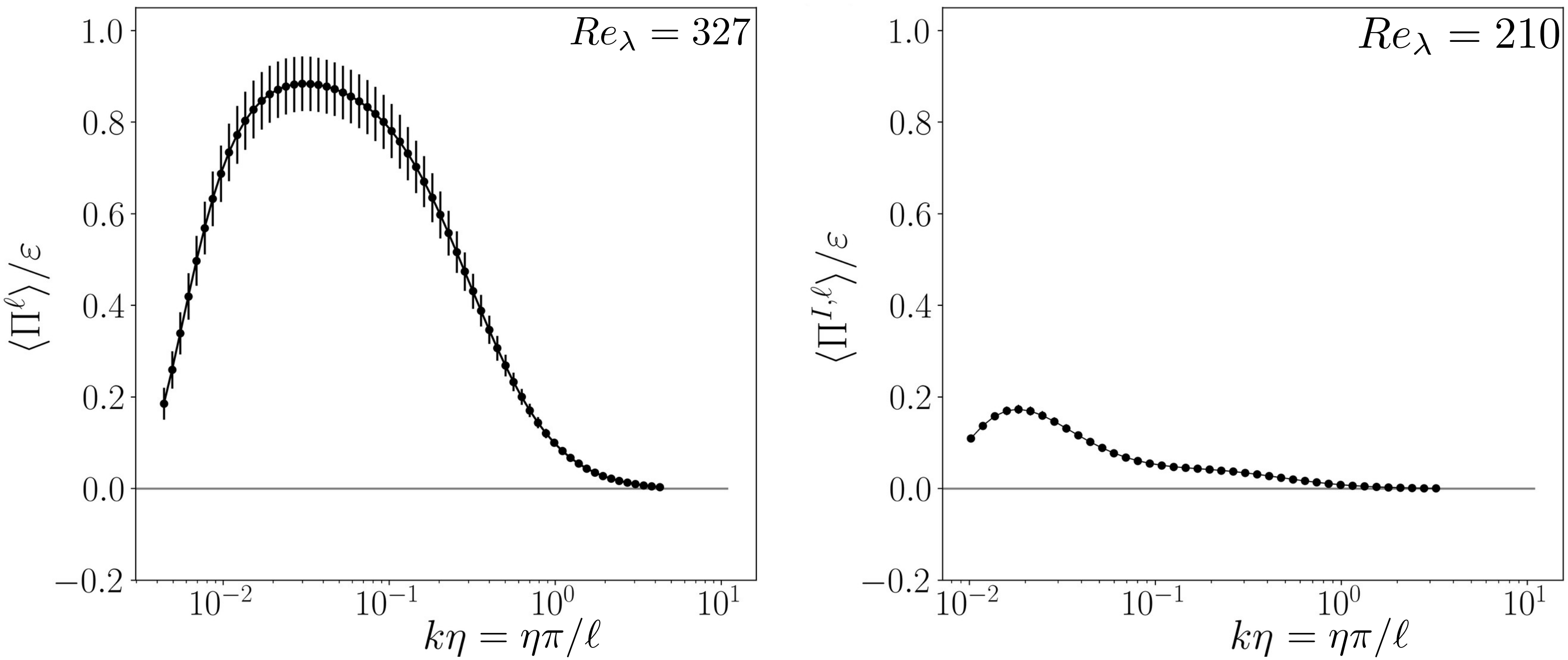
$$\eta = \left(\frac{\nu^3}{\varepsilon_u + \varepsilon_b}\right)^{\frac{1}{4}} \qquad \varepsilon = \varepsilon_u + \varepsilon_b$$

D.C., S. Oughton, P. Johnson, L. Biferale, M. Linkmann (in preparation)

HD versus **MHD**: inertial flux $\boldsymbol{u} \cdot \nabla \boldsymbol{u}$

$$\Pi^{I,\ell} = -\overline{S}^{\ell}_{ij} \tau^{I,\ell}_{ij}$$

Navier-Stokes MHD



→ Understanding the physical processes behind the depletion

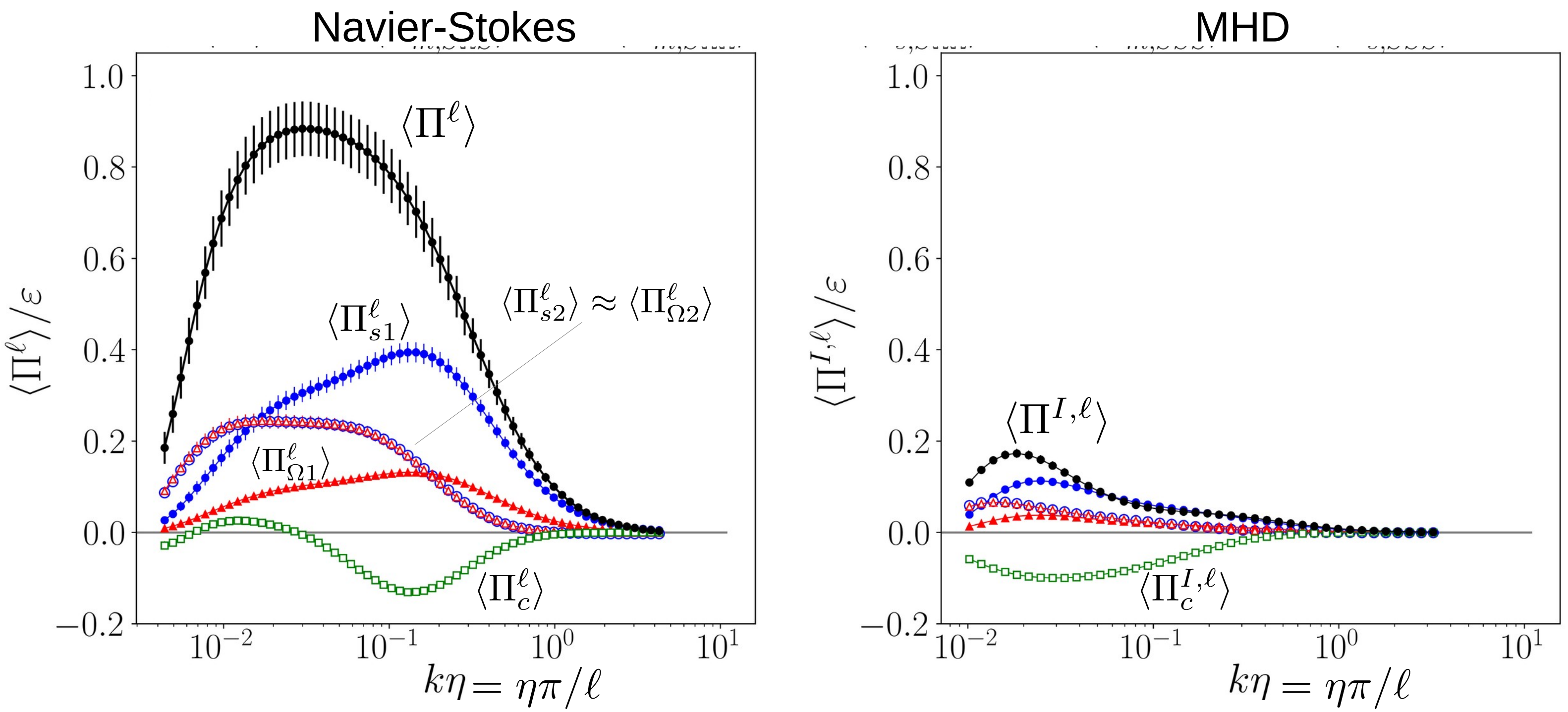
→ vortex-stretching/strain self-amplification → investigating gradient statistics

G. L. Eyink, JFM **549**, 159-190 (2006)
P. Johnson, PRL **124**, 104501 (2020)

$$\overline{S}^{\ell} = \left(\nabla\overline{\boldsymbol{u}}^{\ell} + (\nabla\overline{\boldsymbol{u}}^{\ell})^{t}\right)/2$$
$$\overline{\Omega}^{\ell} = \left(\nabla\overline{\boldsymbol{u}}^{\ell} - (\nabla\overline{\boldsymbol{u}}^{\ell})^{t}\right)/2$$

$$\Pi^{\ell} = \Pi^{\ell}_{s1} + \Pi^{\ell}_{s2} + \Pi^{\ell}_{\Omega 1} + \Pi^{\ell}_{\Omega 2} + \Pi^{\ell}_{c}$$

with $\langle \Pi^{\ell}_{s1}\rangle = 3\langle \Pi^{\ell}_{\Omega 1}\rangle$ via Betchov relation: $-\langle \mathrm{tr}(\overline{S}^{\ell}\overline{S}^{\ell}\overline{S}^{\ell})\rangle = 3\langle \mathrm{tr}(\overline{S}^{\ell}\overline{\Omega}^{\ell}\overline{\Omega}^{\ell})\rangle$



$$\langle \Pi^{\ell}_{s1}\rangle + 3\langle \Pi^{\ell}_{\Omega 1}\rangle = \frac{10}{3}\langle \Pi^{\ell}_{s1}\rangle \approx \frac{1}{2}\langle \Pi^{\ell}\rangle$$

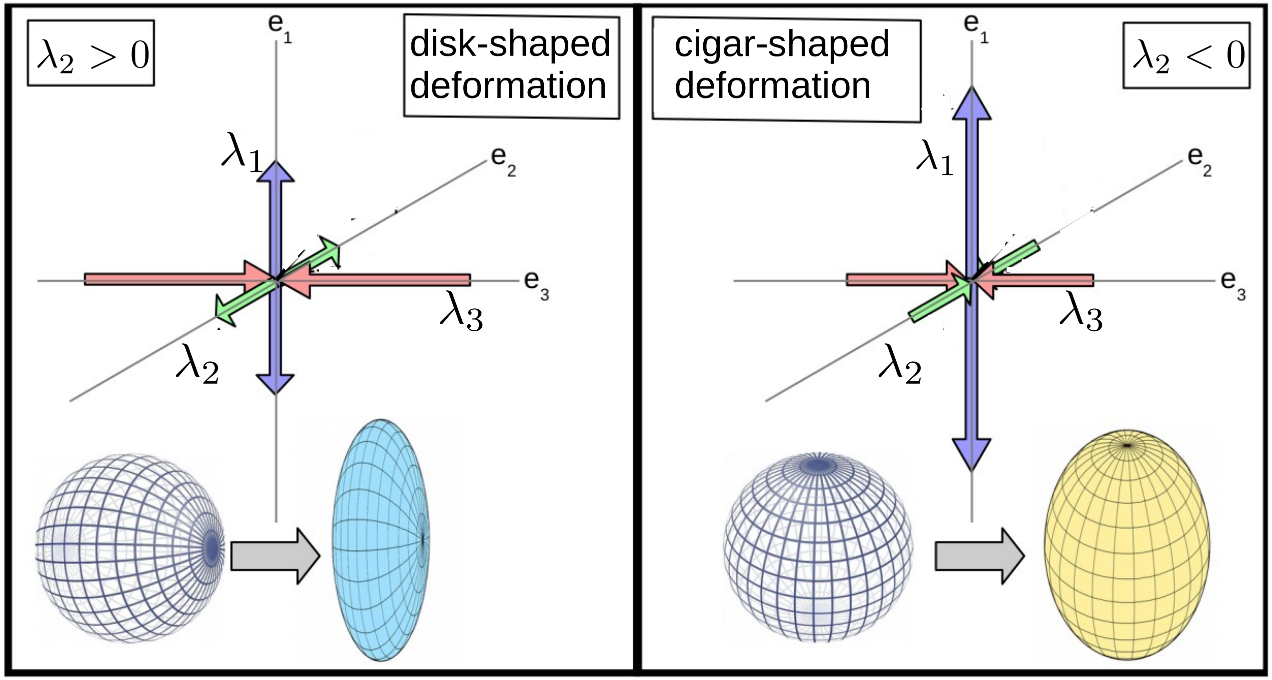
$\langle \Pi^{\ell}_{s1}\rangle = -\ell^2\langle \mathrm{tr}(\overline{S}^{\ell}\overline{S}^{\ell}\overline{S}^{\ell})\rangle = -3\,\ell^2\langle \overline{\lambda}^{\ell}_1\overline{\lambda}^{\ell}_2\overline{\lambda}^{\ell}_3\rangle$ in the ref. frame where $\overline{S}^{\ell}$ is diagonal .

→ We consider he reference frame where S_{ij} is diagonal $S_{ij} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$

→ Fluid sphere deformation: contractile/extensional directions along the eigenvectors

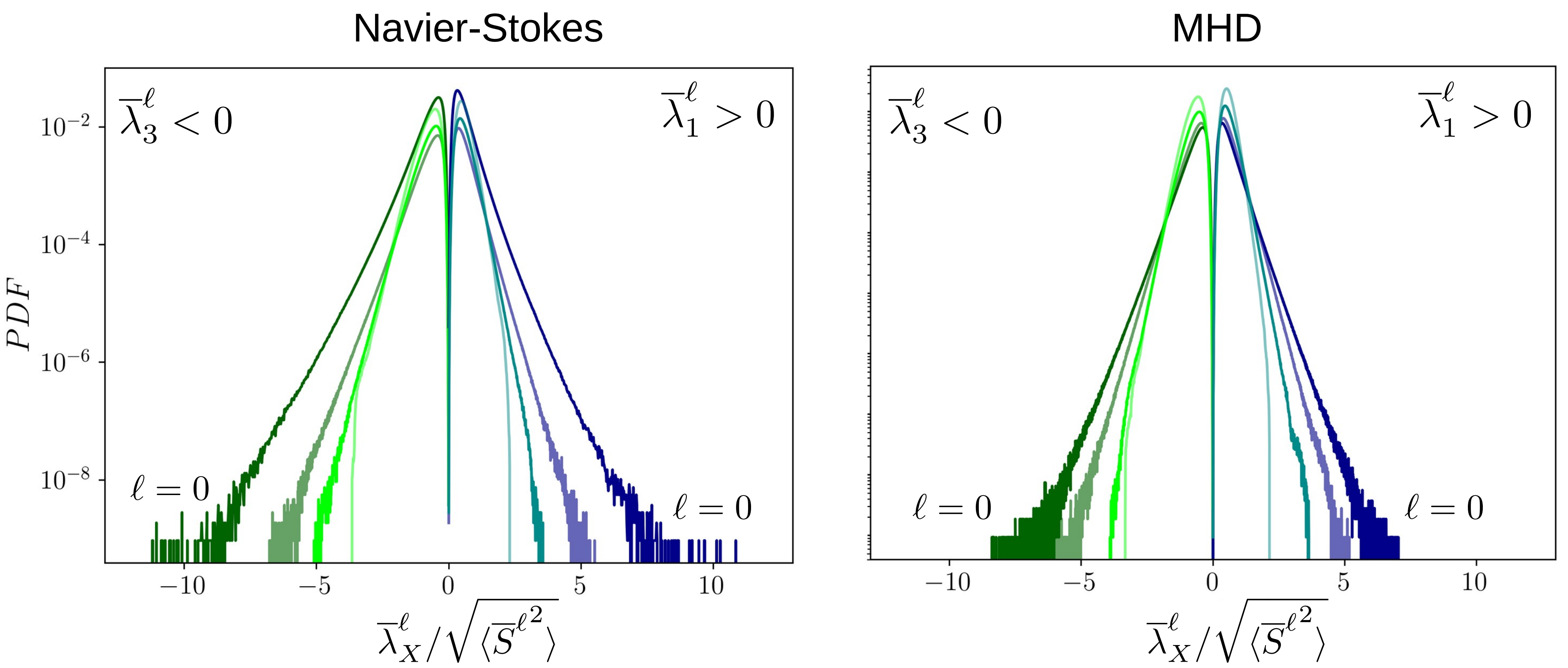
→ Incompressibility i.e. $\nabla \cdot \mathbf{u} = 0$ provides $\lambda_1 + \lambda_2 + \lambda_3 = 0$

→ $\lambda_1 \geq 0,\ \lambda_3 \leq 0$ but $\lambda_2 \lesseqgtr 0$: 2 possible configurations



adapted from
P. Johnson, JFM (2021)

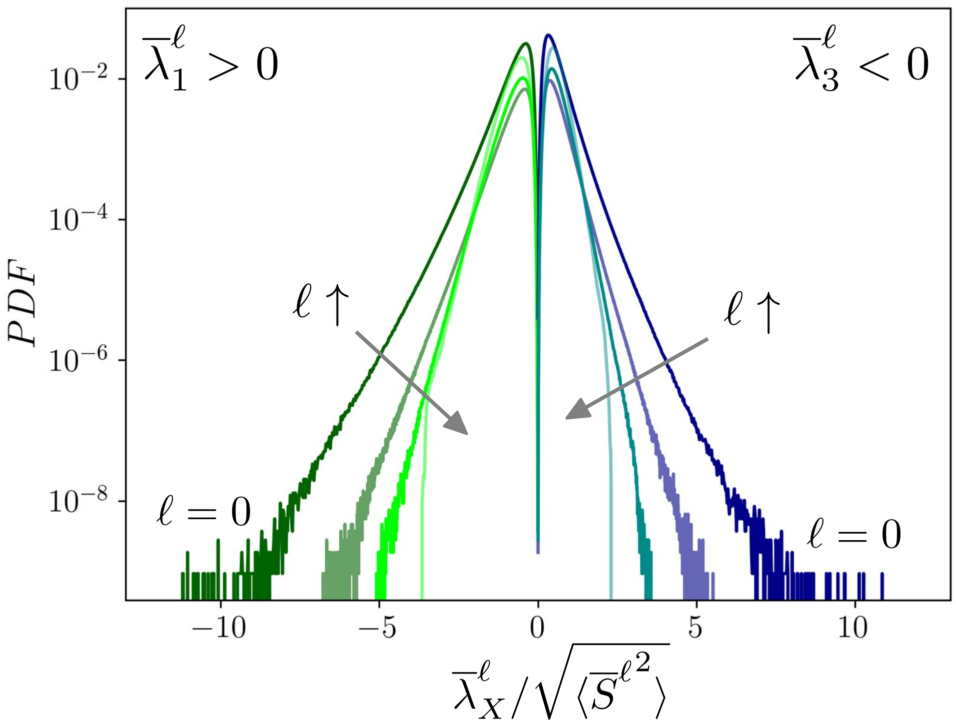
First/third eigenvalues statistics



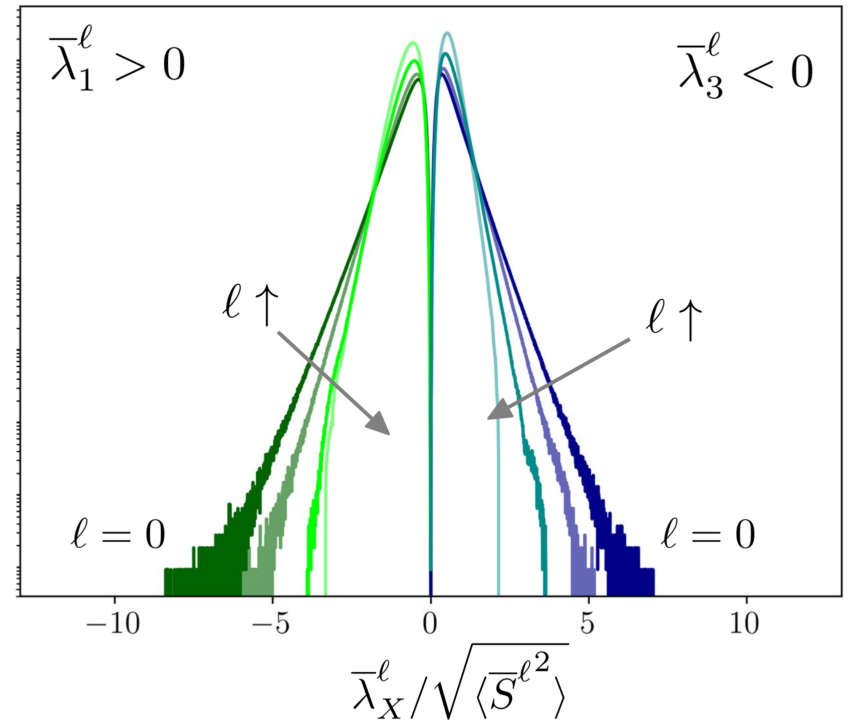
The eigenvalues are sorted: $\overline{\lambda}_1^\ell$ is the largest, $\overline{\lambda}_3^\ell$ the smallest.

First/third eigenvalues statistics

Navier-Stokes



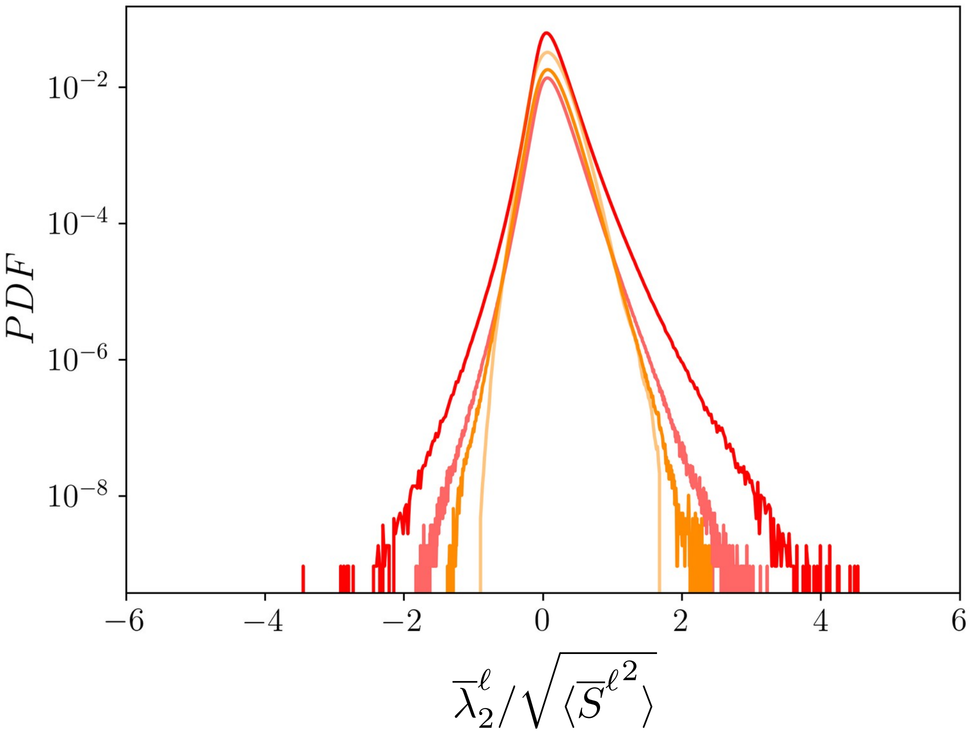
MHD



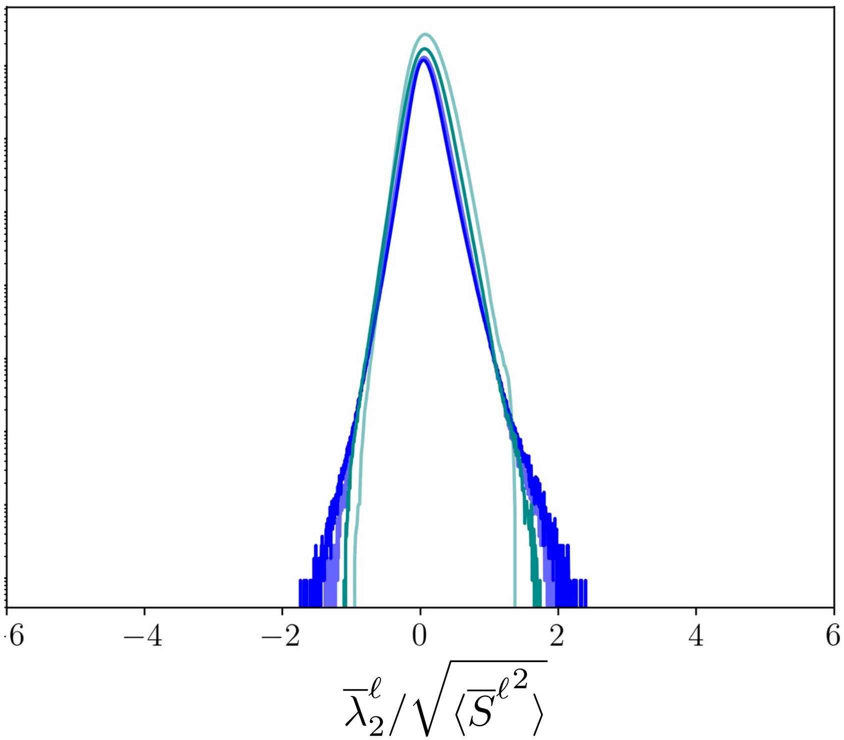
The eigenvalues are sorted: $\overline{\lambda}_1^\ell$ is the largest, $\overline{\lambda}_3^\ell$ the smallest.

Second eigenvalue statistics

Navier-Stokes

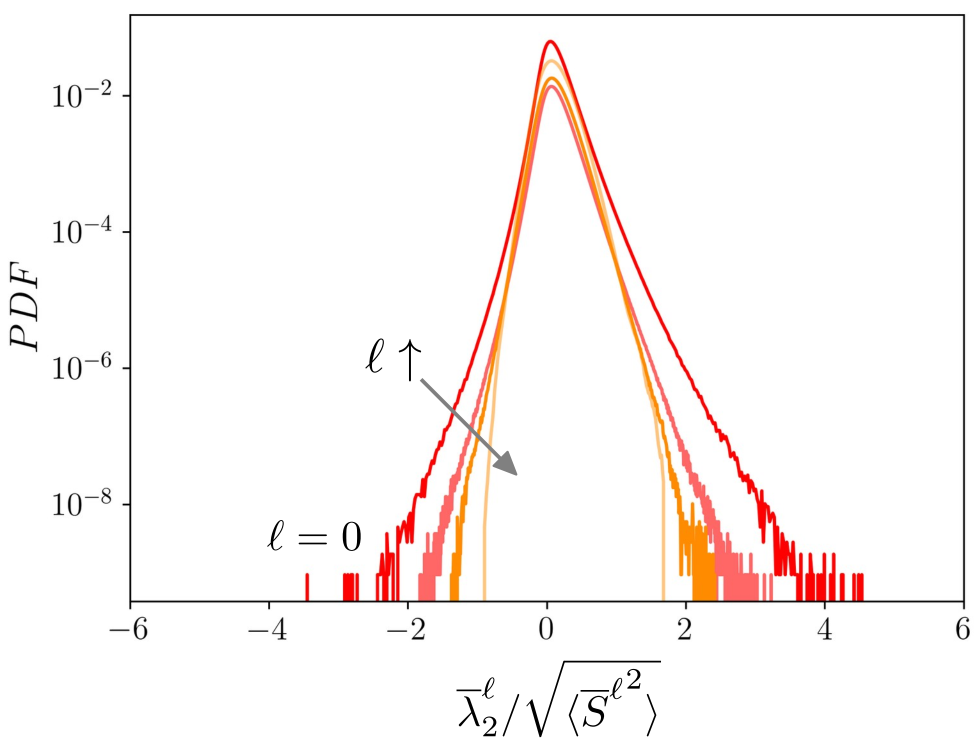


MHD

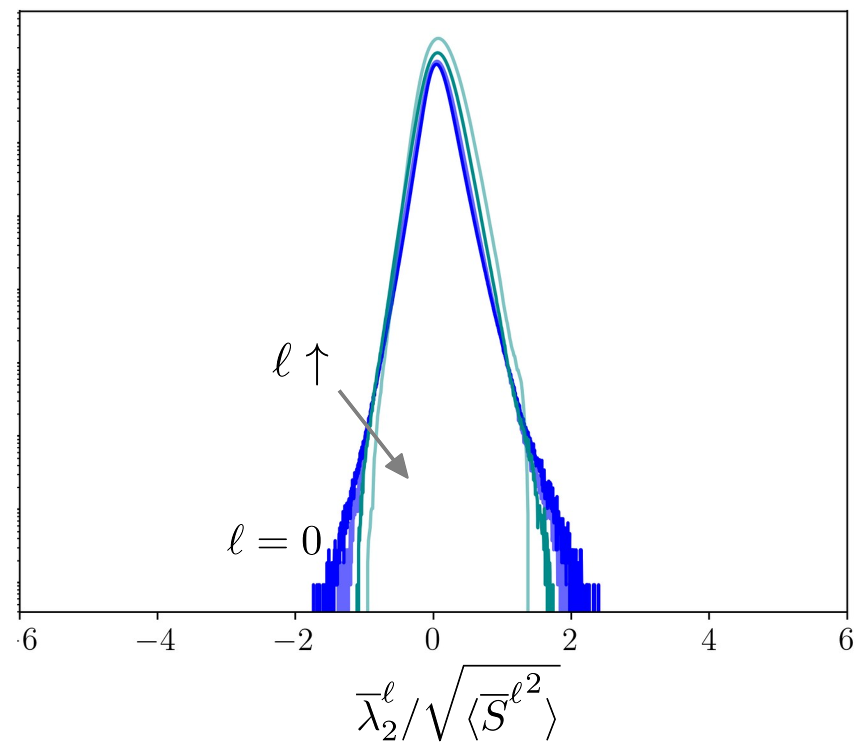


Second eigenvalue statistics

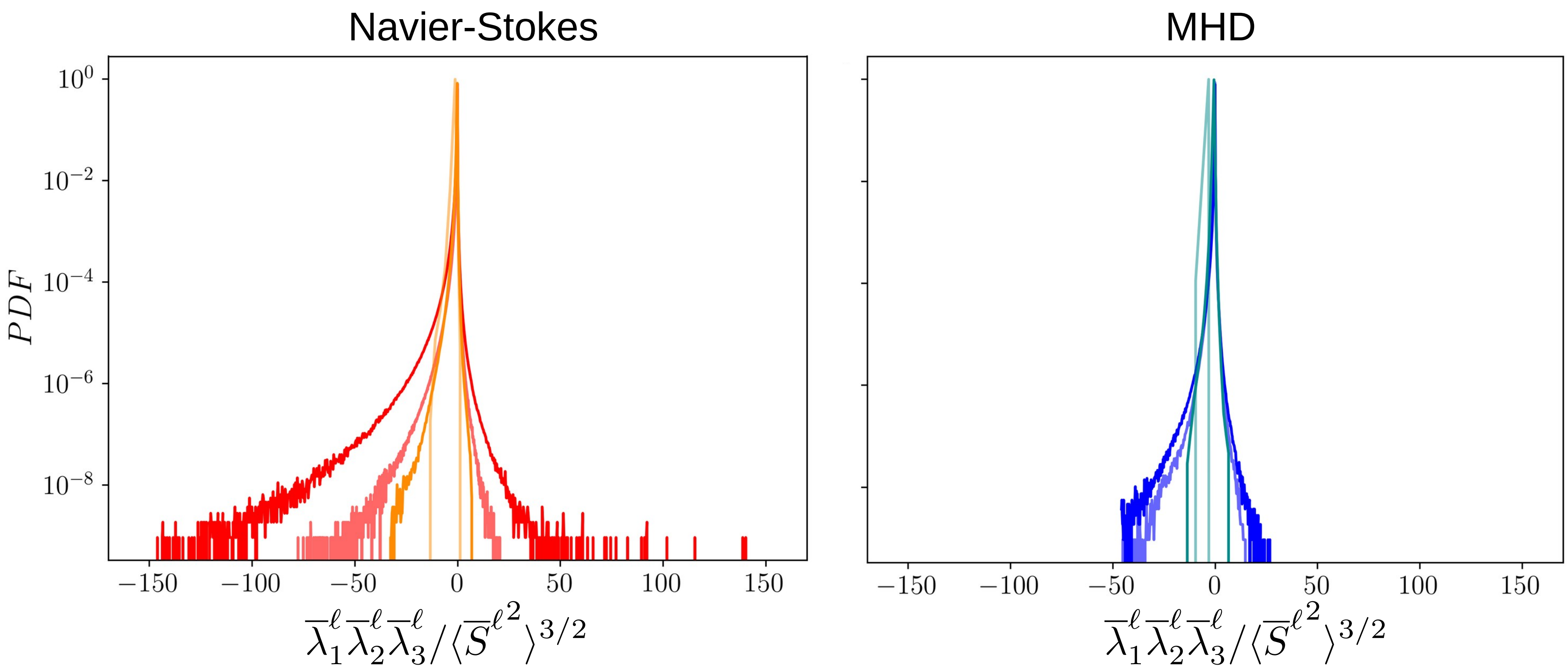
Navier-Stokes



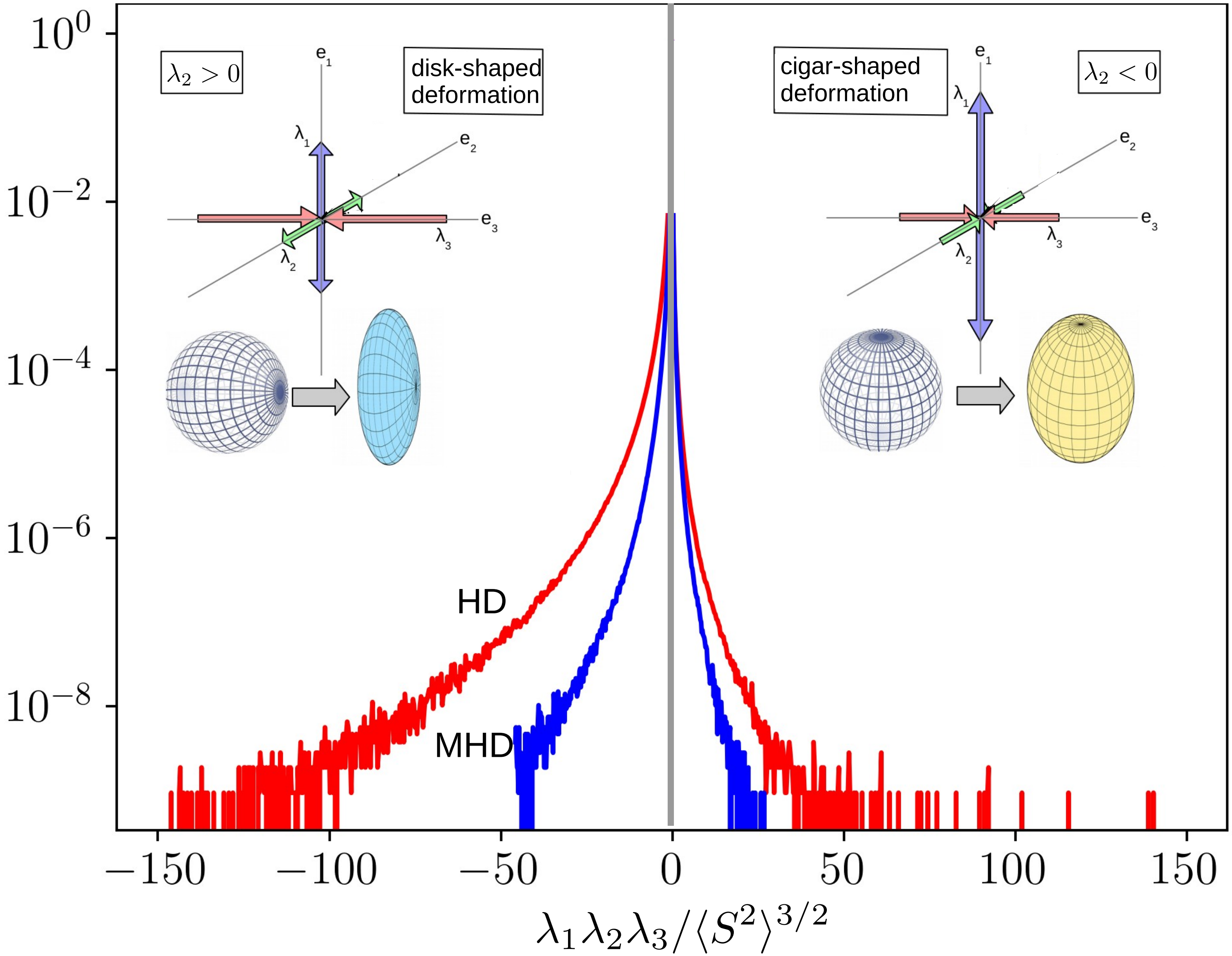
MHD



→ MHD is less scale dependent & more symmetric



$$\langle \Pi_{s1}^{\ell} \rangle = -3\,\ell^2 \langle \overline{\lambda}_1^\ell \overline{\lambda}_2^\ell \overline{\lambda}_3^\ell \rangle$$


$\lambda_2 > 0$
disk-shaped deformation
cigar-shaped deformation
$\lambda_2 < 0$
e1
e2
e3
λ1
λ2
λ3
HD
MHD
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
−150
−100
−50
0
50
100
150
$\lambda_1\lambda_2\lambda_3/\langle S^2\rangle^{3/2}$

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Summary

→ Depletion of vortex stretching in MHD compared to HD

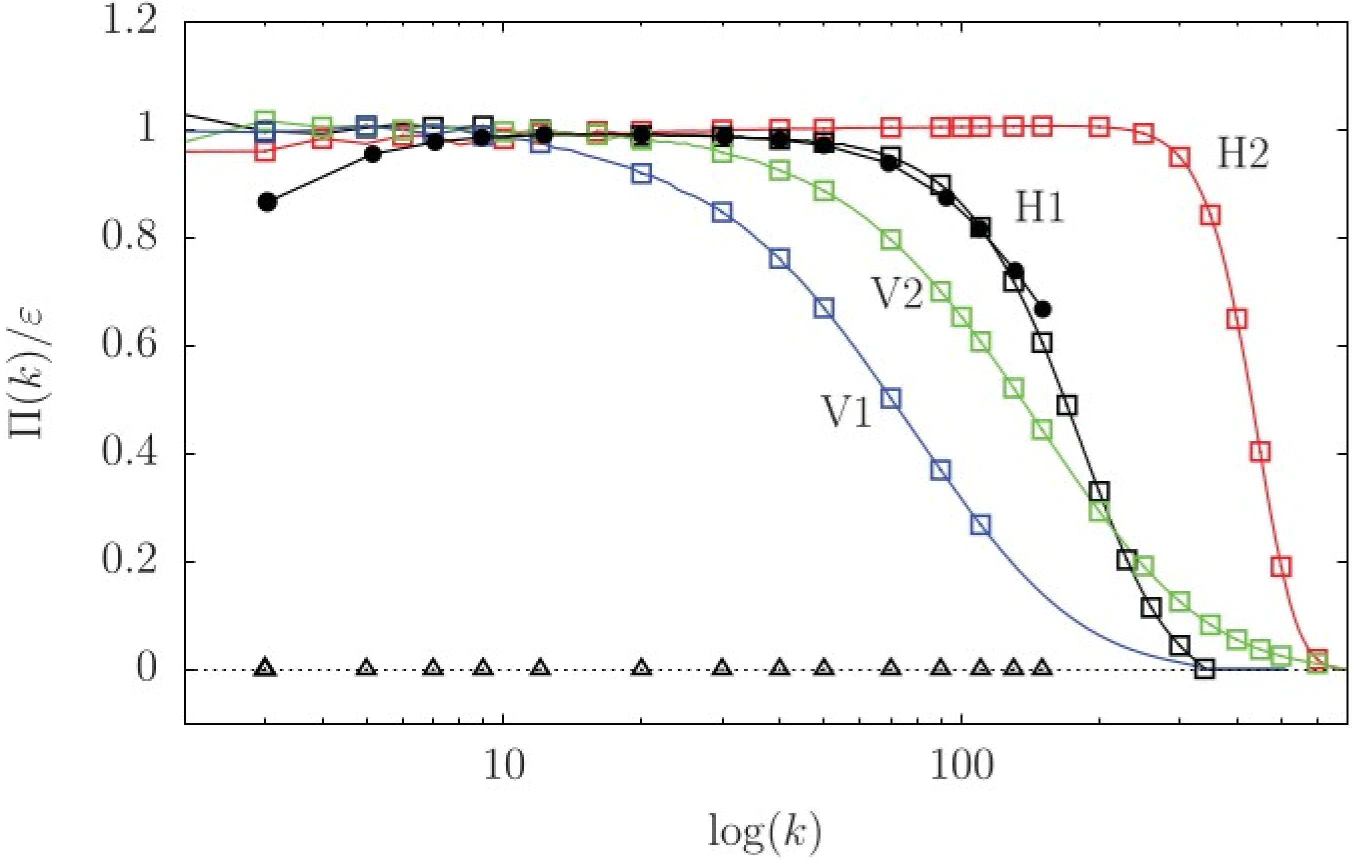
→ HD and MHD have different small-scale flow structures

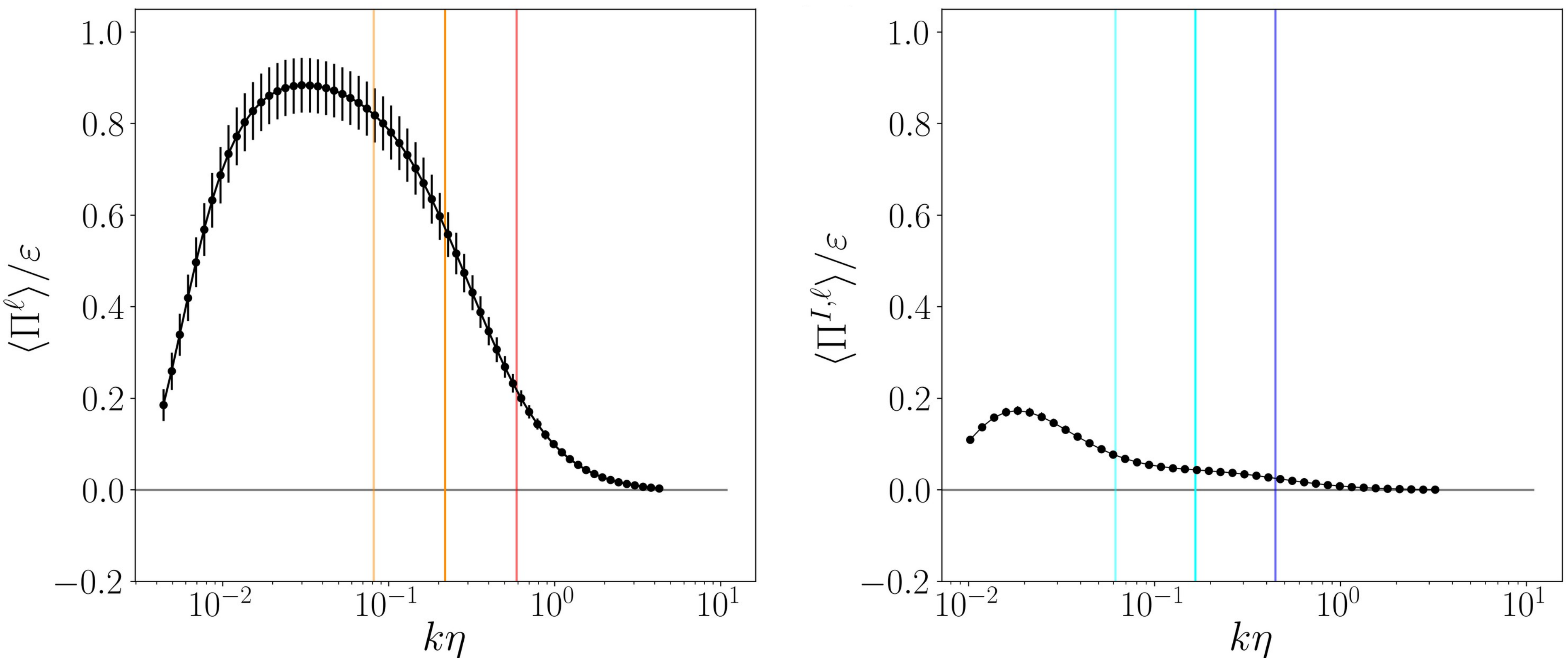
→ the strain-rate tensor eigenvalues distributions are more symmetric in MHD than in HD

→ in HD there is a clear prevalence for disk-shaped deformation of flow structures

→ in MHD there is less difference in likelihood of disk-shaped and cigar-shaped deformation

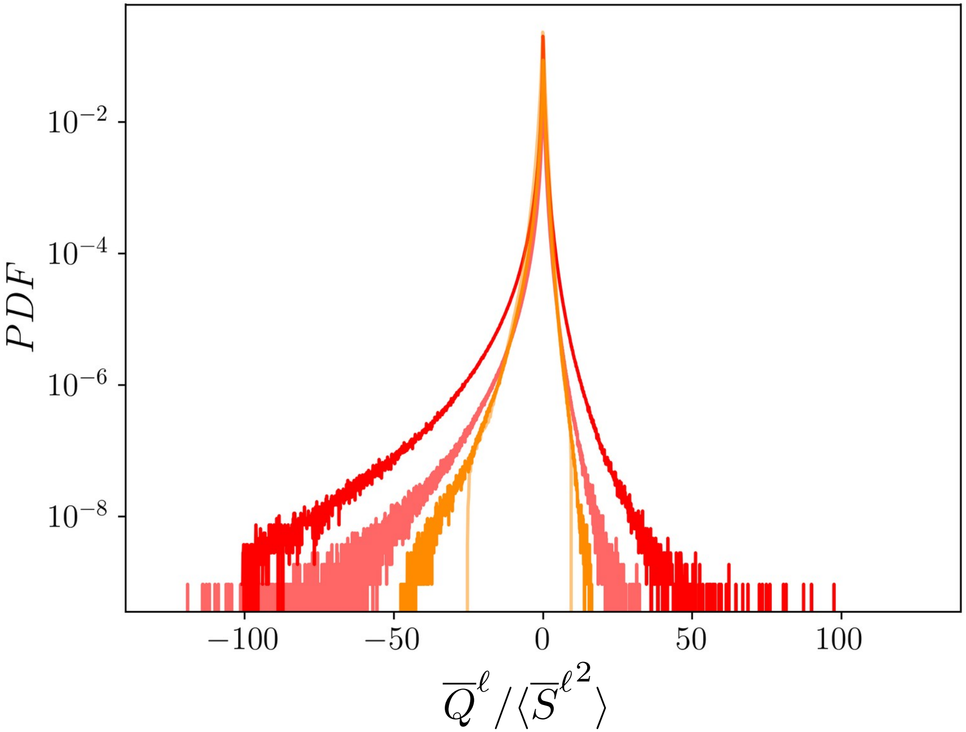
Thank you for your attention

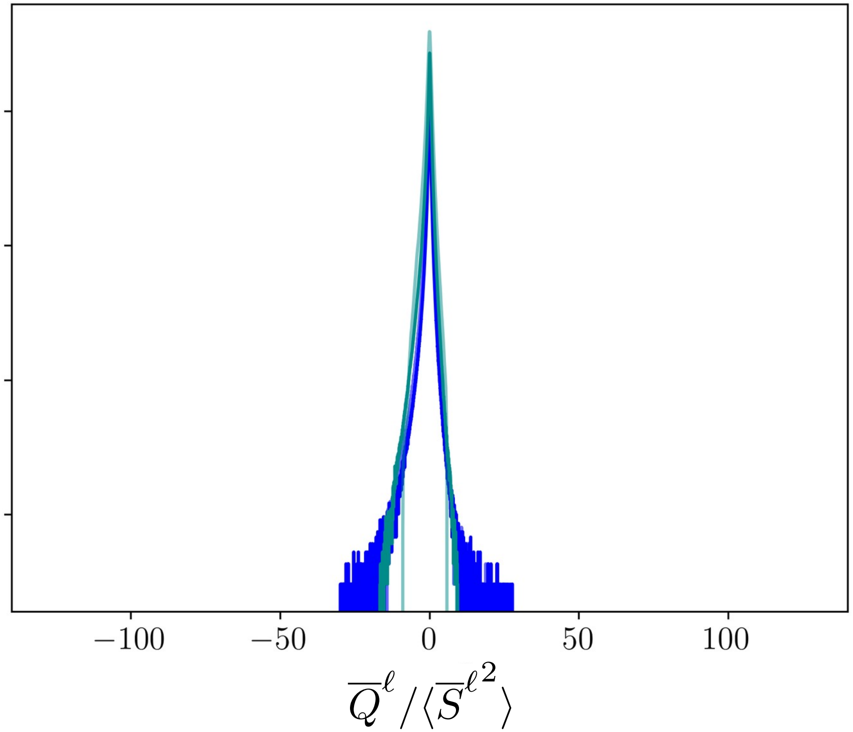

1.2
1
0.8
0.6
0.4
0.2
0
10
100
log(k)
Π(k)/ε
H2
H1
V2
V1


$\langle \Pi^{\ell} \rangle / \varepsilon$
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
10^{-2}
10^{-1}
10^{0}
10^{1}
$k\eta$
$\langle \Pi^{I,\ell} \rangle / \varepsilon$
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
10^{-2}
10^{-1}
10^{0}
10^{1}
$k\eta$

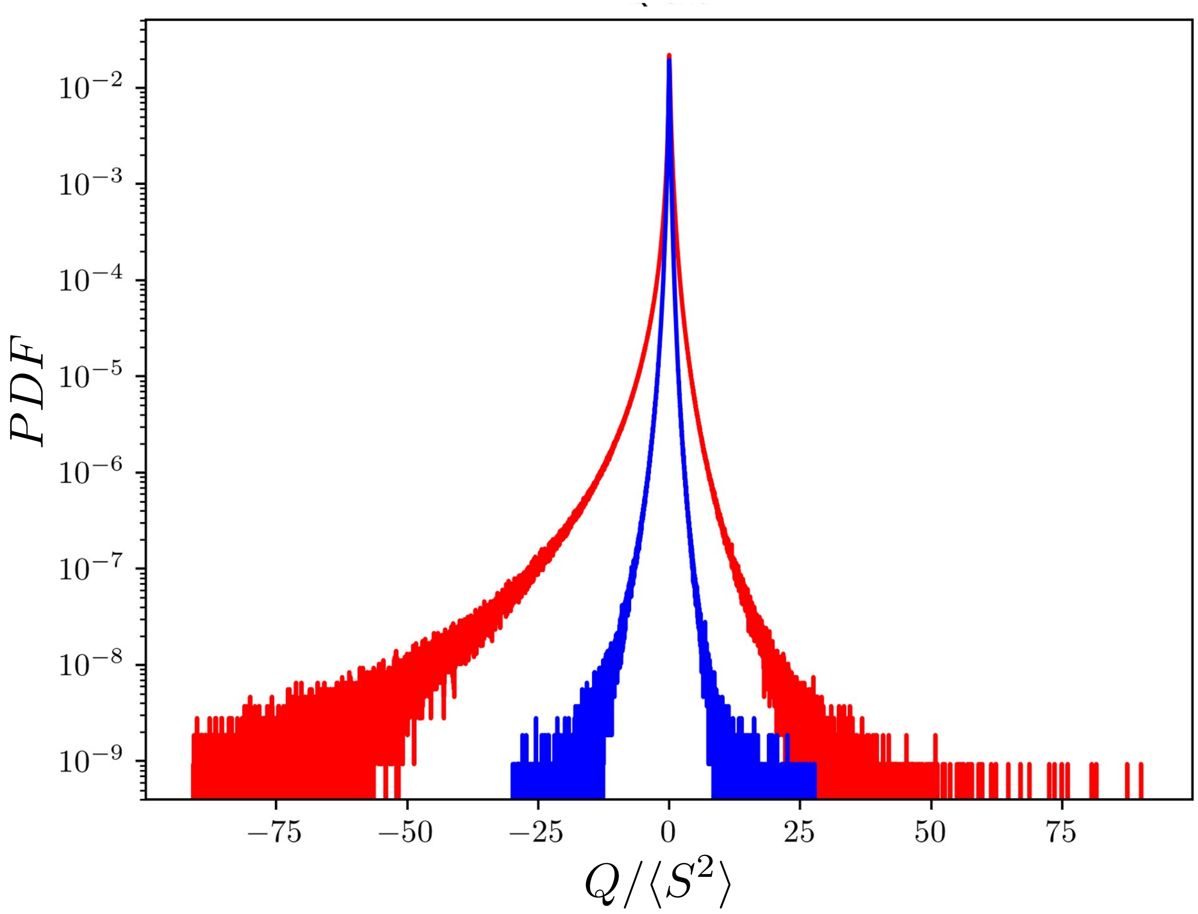
Q criterion statistics

$$Q = \frac{1}{2}\left(S^2 - \frac{1}{2}\omega^2\right)$$





$$Q = \frac{1}{2}\left(S^2 - \frac{1}{2}\omega^2\right)$$



→ More vorticity in HD