Exploring Turbulence: A Personal Journey through Critical Behavior, Anomalous Scaling, and Data-Driven Approaches

$$\partial_{\hat{t}}\hat{v} + \hat{v}\cdot\hat{\partial}\hat{v} = -\hat{\partial}\hat{p} + \frac{1}{Re}\hat{\Delta}\hat{v} + \hat{F}_{mech}$$

Leonardo da Vinci (**~ 1500**): "doue la turbolenza de l'aqua si genera **[injected]**; doue la turbolenza dell aqua si mantiene **[advected/conserved]** plugho; doue la turbolenza dell acqua si posa **[dissipated]**"

-OUT OF EQUILIBRIUM (INJECTED-DISSIPATED)
-NONLINEAR (ADVECTED/CONSERVED)

U. Chicago 2023

Luca Biferale Dept. Physics & INFN - University of Rome 'Tor Vergata' biferale@roma2.infn.it

Exploring Turbulence: A Personal Journey through Critical Behavior, Anomalous Scaling, and Data-Driven Approaches

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- -OUT OF EQUILIBRIUM (INJECTED-DISSIPATED) -> DISSIPATIVE ANOMALY
- -(STRONGLY) NONLINEAR (ADVECTED/CONSERVED)
- -NON-GAUSSIAN (EXTREME EVENTS +ANOMALOUS SCALING)
- -MULTI-SCALE (TRILLION OF DEGREES OF FREEDOM)
- -IMPORTANT FOR MANY REAL WORLD APPLICATIONS

U. Chicago 2023

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FARE RICERCA IN ITALIA
FRAMEWORK PER L'ATTRAZIONE E IL RAFFORZAMENTO DELLE ECCELLENZE PER LA RICERCA IN ITALIA
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TURBULENCE OR TURBULENCES?

MASS X ACCELERATION = INTERNAL FORCES + EXTERNAL FORCES

EULERIAN

$$\rho[\partial_t v + v \cdot \partial v] = -\partial p + \nu \Delta v + \boldsymbol{g}\theta + F(\boldsymbol{B}, \boldsymbol{B}) + 2\Omega \times v + \hat{F}_{mech}$$

$$\partial_t \theta + \boldsymbol{v} \cdot \boldsymbol{\partial} \theta = \chi \partial^2 \theta$$ ← TEMPERATURE

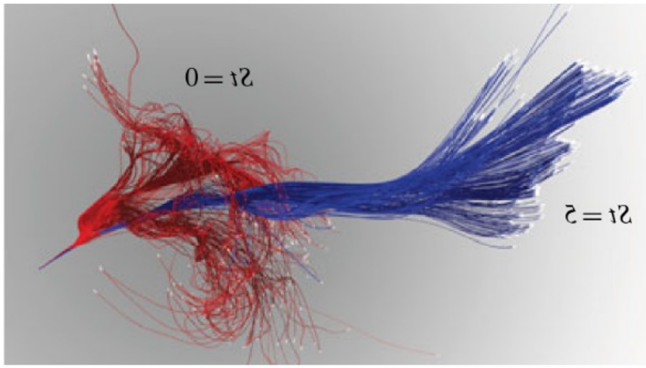
$$\partial_t \boldsymbol{B} + \boldsymbol{v} \cdot \boldsymbol{\partial} \boldsymbol{B} = \boldsymbol{B} \cdot \boldsymbol{\partial} \boldsymbol{v} + \chi \partial^2 \boldsymbol{B}$$ ← MAGNETIC FIELD

$$\partial \cdot v = 0$$

+ BOUNDARY CONDITIONS: (2D, 3D, THIN/THICK LAYERS ETC...)

LAGRANGIAN

$$\dot{X}(t) = v(X(t), t)$$



A. Alexakis, L. B. , Cascades and transitions in turbulent flows. Phys. Rep. 767, 1–101 (2018)

THE HYDROGEN ATOM OF TURBULENCE (not solvable though!)

$$\partial_{\hat{t}}\hat{v} + \hat{v}\cdot\hat{\partial}\hat{v} = -\hat{\partial}\hat{p} + \frac{1}{Re}\hat{\Delta}\hat{v} + \hat{F}_{mech}$$

+PERIODIC BOUNDARY CONDITIONS

ENERGY INJECTION

ENERGY DISSIPATION

$$\begin{cases} \hat{t} = t/t_0 \\ \hat{x} = x/l_0 \\ \hat{v} = v/v_0 \end{cases}$$

$$\hat{F} \sim F\ell_0/v_0^2$$

$$Re \sim \frac{v\partial v}{\nu\partial^2 v} \qquad Re = \frac{l_0 v_0}{\nu}$$

Reynolds number ~ (Non-Linear)/(Linear terms)

$$Re \to \infty$$

Fully Developed Turbulence:

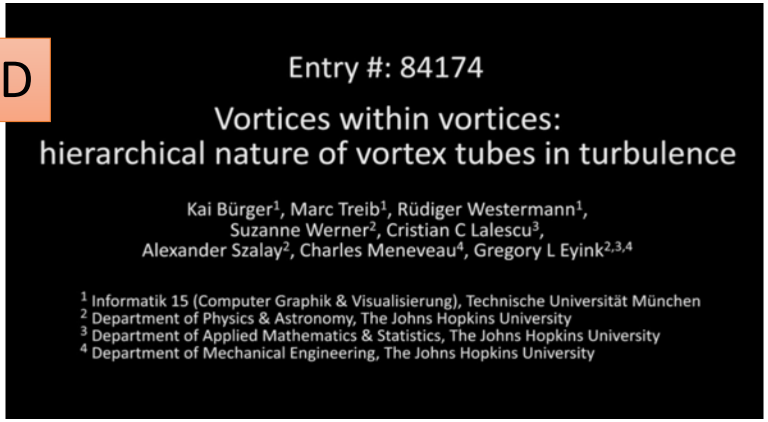
1. Strongly non-linear & non-perturbative system

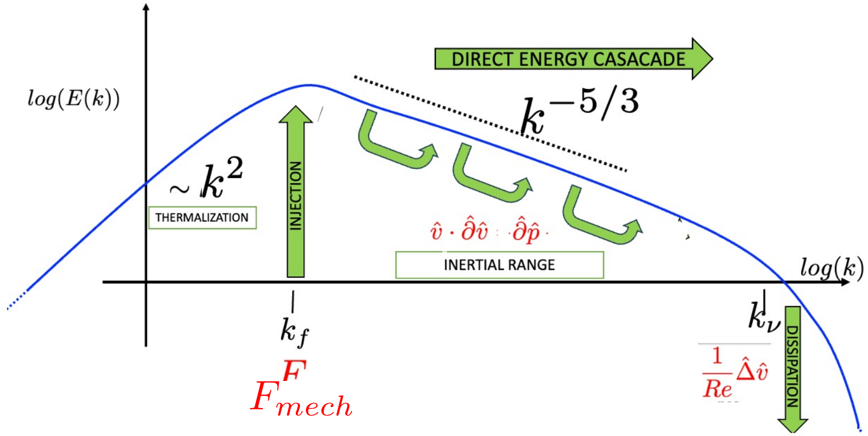
DIFFICULT TO DISSIPATE? WAIT AND SEE…

BASIC FUNDAMENTAL OPEN PROBLEMS ABOUT GLOBAL AND LOCAL PROPERTIES

1. WE DO NOT CONTROL THE MEAN ENERGY FLUX: DOES THE INJECTED ENERGY FLOW TOWARDS SMALL-SCALES OR TOWARDS LARGE-SCALES (OR BOTH)?
2. WE DO NOT CONTROL THE SCALE-BY-SCALE MEAN ENERGY SPECTRUM
3. WE DO NOT CONTROL NEITHER EULERIAN NOR LAGRANGIAN FLUCTUATIONS AROUND THE MEAN GLOBAL PROPERTIES

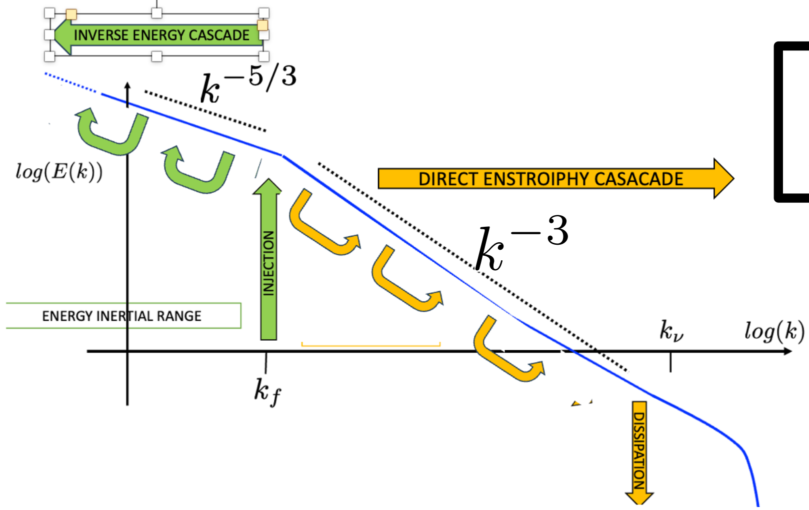
3D





3D

2D



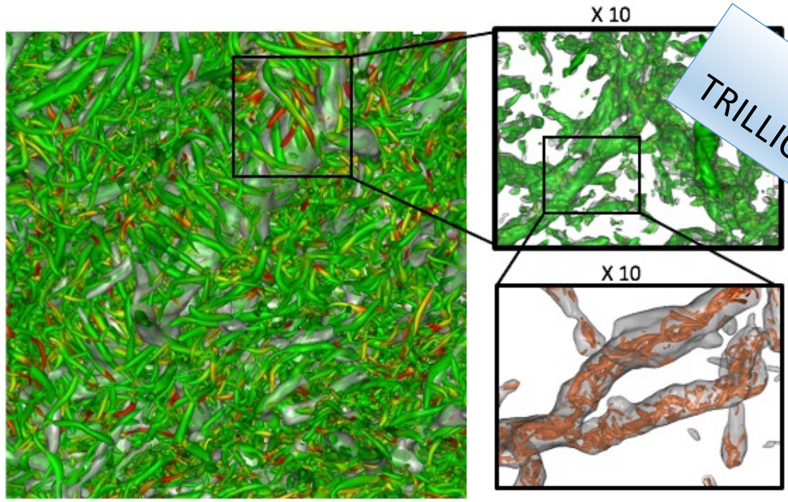
2 invariants (positive definite)

$$\Omega = \int d^3x\, \omega \cdot \omega \quad E = \int d^3x\, \mathbf{v} \cdot \mathbf{v}$$

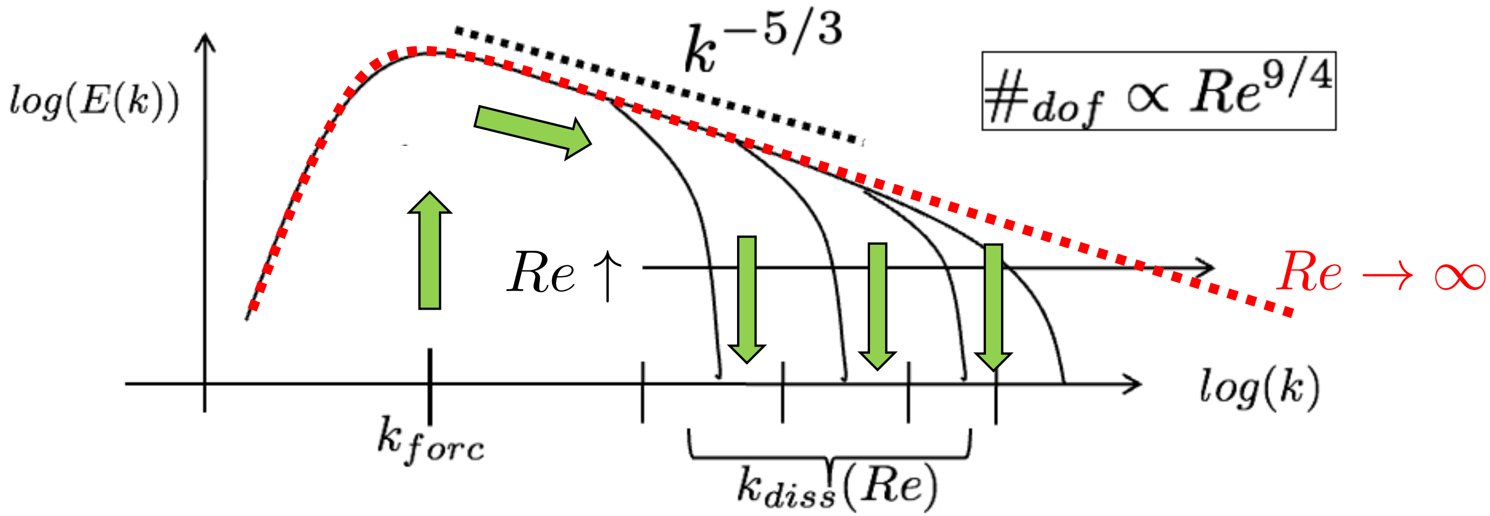
2D

3D – DIRECT CASCADE

$$\partial_{\hat{t}}\hat{v} + \hat{v}\cdot\hat{\partial}\hat{v} = -\hat{\partial}\hat{p} + \frac{1}{Re}\hat{\Delta}\hat{v} + \hat{F}_{mech}$$



1. MULTISCALE !
TRILLION OF DEGREES OF FREEDOM



3D – DIRECT CASCADE

$$\hat{\partial}_{\hat{t}}\hat{v} + \hat{v}\cdot\hat{\partial}\hat{v} = -\hat{\partial}\hat{p} + \frac{1}{Re}\hat{\Delta}\hat{v} + \hat{F}_{mech}$$

76th Annual Meeting of the Division of Fluid Dynamics
Sunday–Tuesday, November 19–21, 2023; Washington, DC

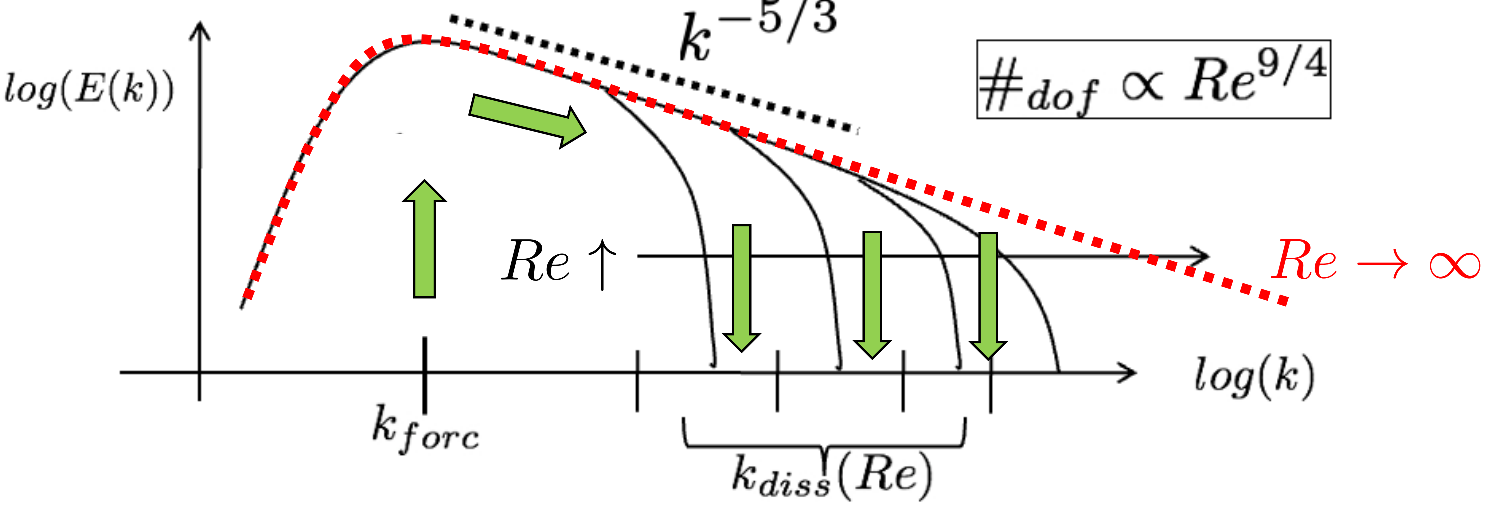
Session T02: Turbulence: DNS
4:25 PM–5:56 PM, Monday, November 20, 2023
Room: Ballroom B

Chair: Robert Moser, University of Texas at Austin

Abstract: T02.00005 : Turbulence simulations at grid resolution up to 32768^3 enabled by Exascale computing*
5:17 PM–5:30 PM

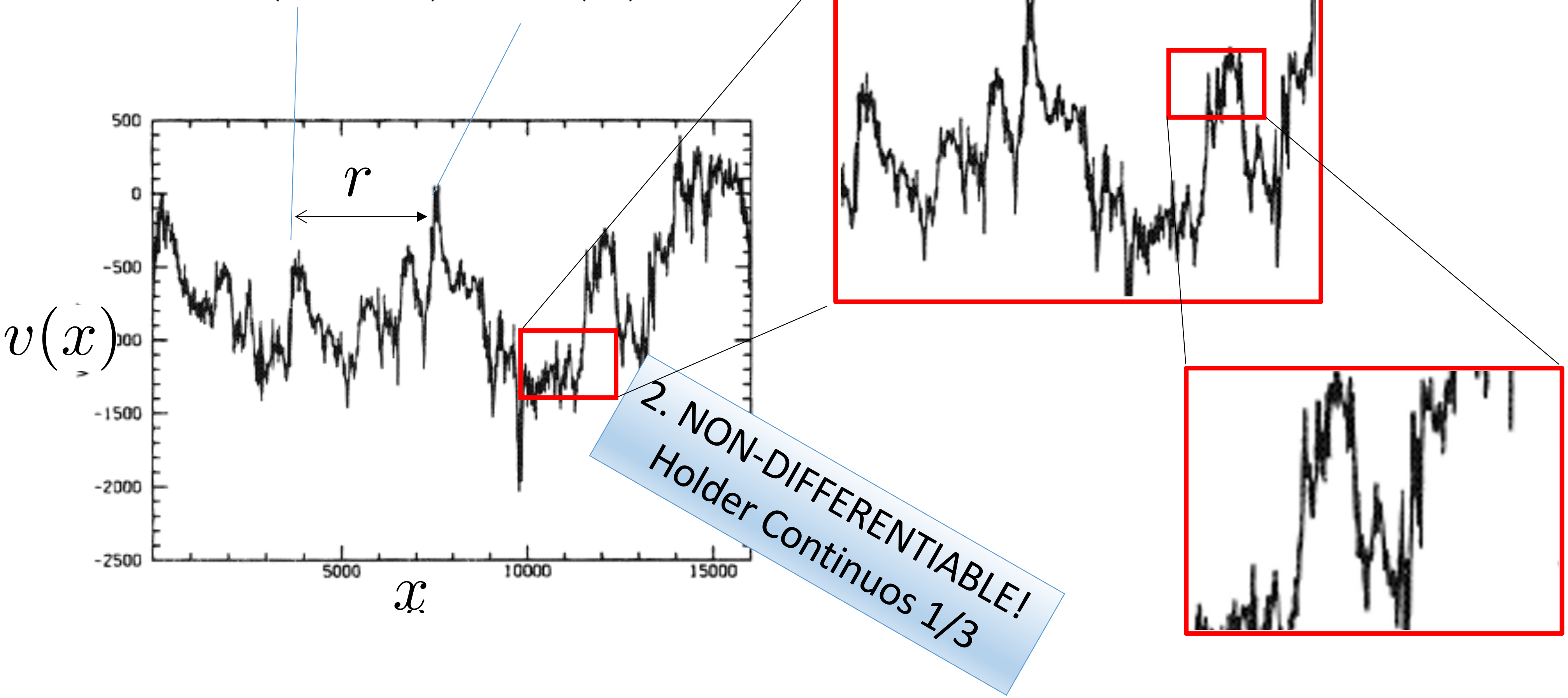
Presenter:
Pui-Kuen (P.K) Yeung
(Georgia Institute of Technology)

32768^3 ~ 35 TRILLION GRID POINT
1 CONF ~ 1 PETABYTE



$$\partial_{\hat{t}}\hat{v} + \hat{v}\cdot\hat{\partial}\hat{v} = -\hat{\partial}\hat{p} + \frac{1}{Re}\hat{\Delta}\hat{v} + \hat{F}$$

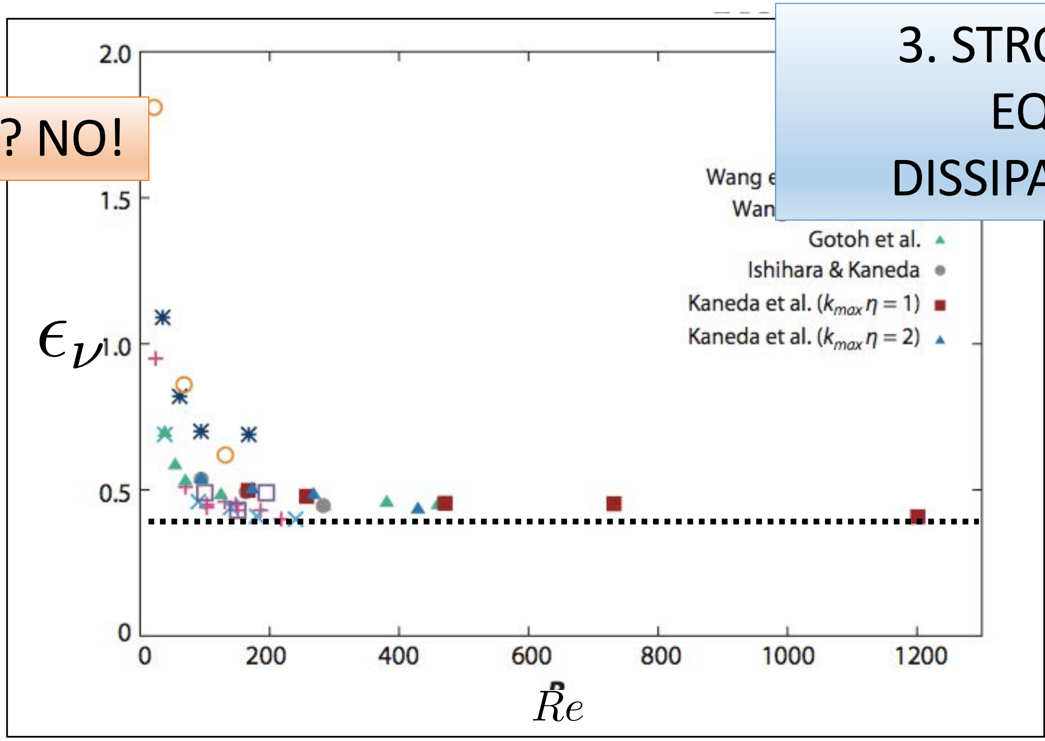
$$\delta_r v = v(x+r) - v(x) \sim r^{\frac{1}{3}}$$



$$\partial_{\hat{t}}\hat{v} + \hat{v}\cdot\hat{\partial}\hat{v} = -\hat{\partial}\hat{p} + \frac{1}{Re}\hat{\Delta}\hat{v} + \hat{F}$$

DIFFICULT TO DISSIPATE? NO!

3. STRONGLY OUT-OF-EQUILIBRIUM DISSIPATIVE ANOMALY



$$\lim_{Re\to\infty} \epsilon_\nu = \lim_{Re\to\infty} \frac{1}{Re}\langle \hat{v}\Delta\hat{v}\rangle \to O(1)$$

NO ROOM FOR QUASI-EQUILIBRIUM STAT MECH!

T. Ishihara, T. Gotoh, Y. Kaneda, Study of high–Reynolds number isotropic turbulence by direct numerical simulation, Annu. Rev. Fluid Mech. 41 (2009) 165–180.

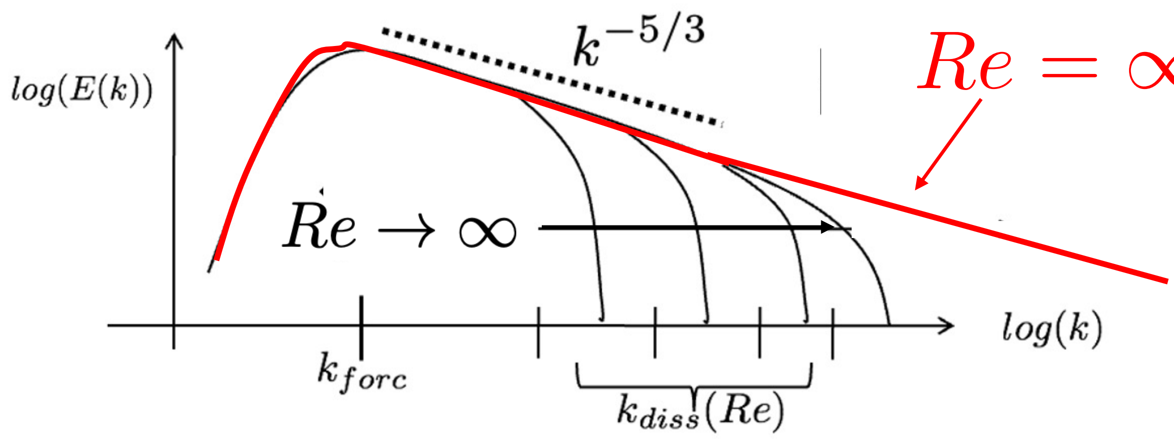
G. Eyink & K.R. Sreenivasan, "Onsager and the theory of hydrodynamic turbulence," Rev. Mod. Phys. 78 87-135 (2006)

$$\partial_{\hat{t}}\hat{v} + \hat{v}\cdot\hat{\partial}\hat{v} = -\hat{\partial}\hat{p}$$

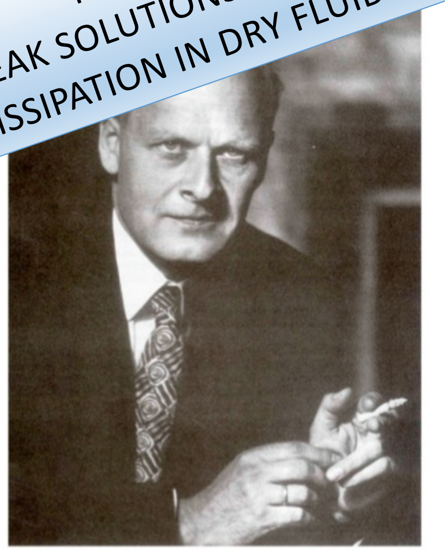
Statistical Hydrodynamics.

L. Onsager
New Haven, Conn.

NUOVO CIMENTO, 1949



THERE EXIST
DISSIPATIVE WEAK SOLUTIONS OF EULER EQUATIONS:
DISSIPATION IN DRY FLUIDS !



It is of some interest to note that in principle, turbulent dissipation as described could take place just as readily without the final assistance by viscosity. In the absence of viscosity, the standard proof of the conservation of energy does not apply, because the velocity field does not remain differentiable! In fact it is possible to show that the velocity field in such "ideal" turbulence cannot obey any Lipschitz condition of the form

(26) $$v(x+r) - v(x) \sim r^h \qquad h \leq 1/3$$

Annals of Mathematics 188 (2018), 871–963
https://doi.org/10.4007/annals.2018.188.3.4

A proof of Onsager's conjecture

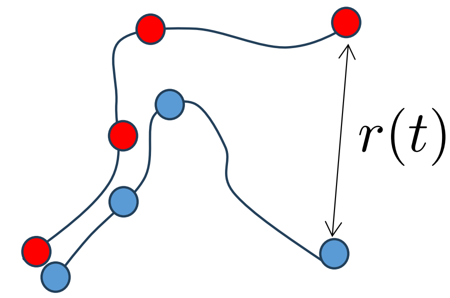
By Philip Isett

RICHARDSON DISPERSION: SPONTANEOUS STOCHASTICITY OF ADVECTED PARTICLES

LAGRANGIAN

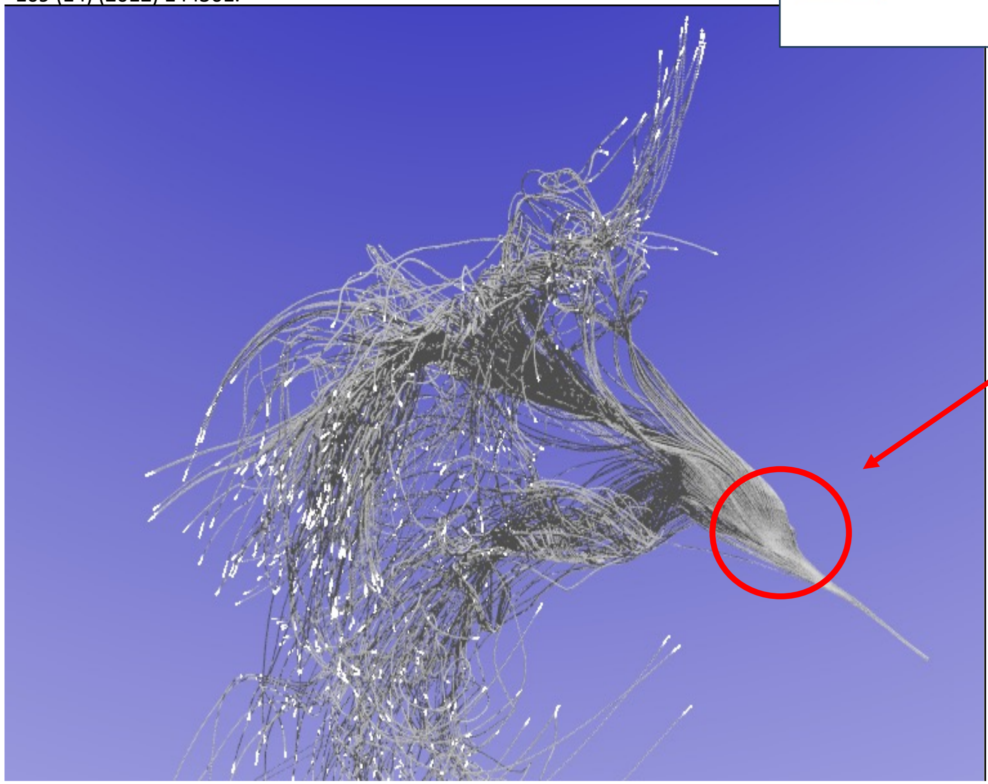
$$\dot{X}(t) = v(X(t), t)$$

$$r(t) = X_1(t) - X_2(t)$$



-G. Falkovich, K. Gawedzki, M. Vergassola, Particles and fields in fluid turbulence, Rev. Modern Phys. 73 (4) (2001) 913

-R. Scatamacchia, L. B., F. Toschi, Extreme events in the dispersions of two neighboring particles under the influence of fluid turbulence, Phys. Rev. Lett. 109 (14) (2012) 144501.



$$\frac{1}{2}\frac{d}{dt}\langle r^2\rangle = \langle r\delta_r v\rangle \sim \epsilon^{\frac{1}{3}} r^{\frac{4}{3}}$$

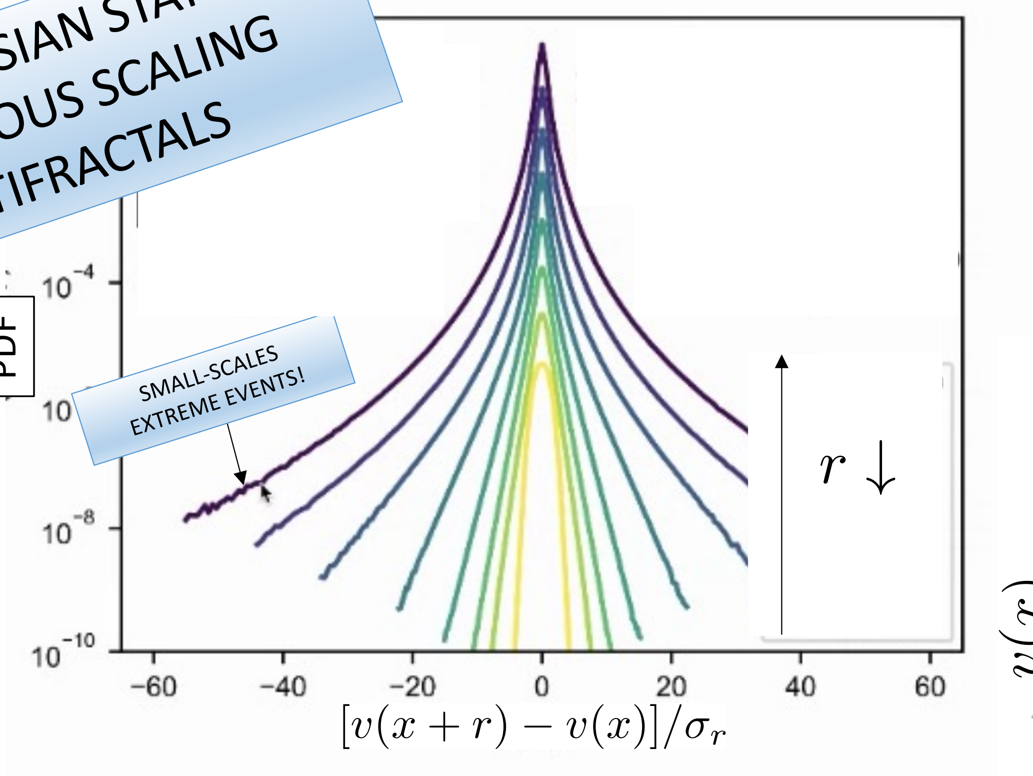
$$\langle r(t)^2\rangle \sim r(0)^2 + g\epsilon t^3$$

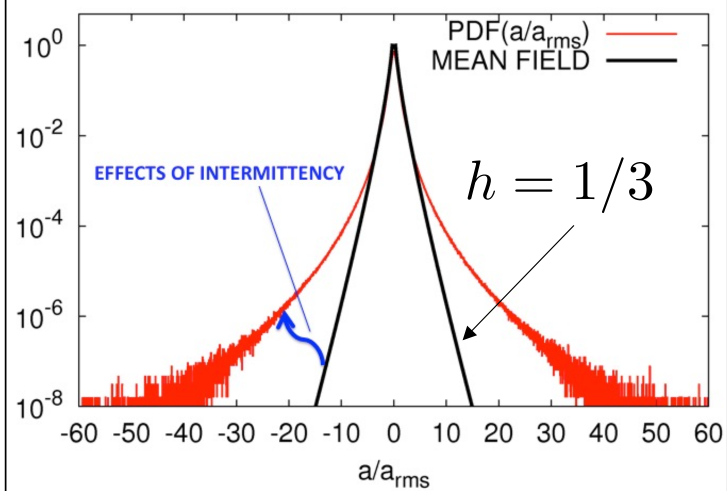
$$P(r,t) \sim \frac{r^2}{t^{\frac{9}{2}}} exp\left[-b r^{\frac{2}{3}}/t\right]$$

TWO PARTICLES SEPARATE EVEN IF STARTING ON THE SAME POSITION!

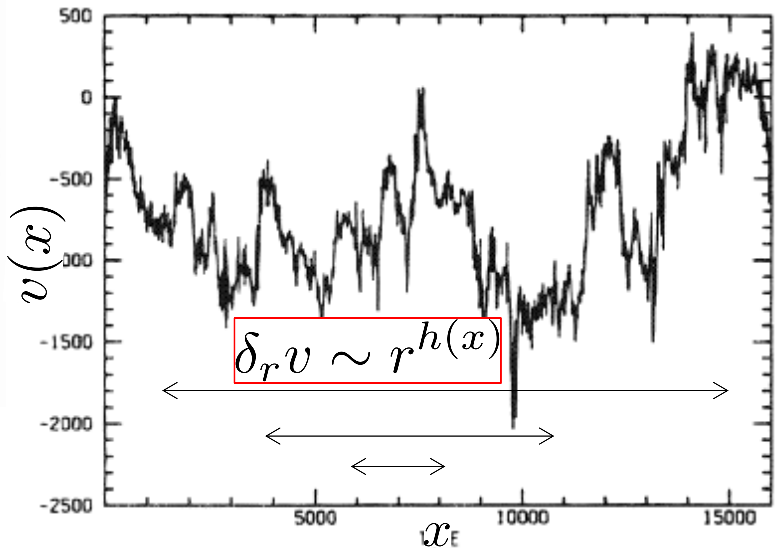
$$\delta_r v \sim r^{h(x)} \longleftrightarrow h(x) \in [h_m, h_M]$$

4. NON-GAUSSIAN STATISTICS ANOMALOUS SCALING MULTIFRACTALS



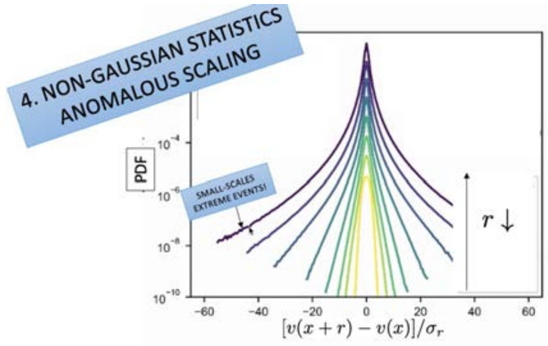


ACCELERATION PROBABILITY DISTRIBUTION FUNCTION (PDF) AT RE ~ 10^5 [Bi04] COMPARED WITH THE PREDICTION FROM MEAN FIELD (KOLMOGOROV THEORY)



Bentkamp, L, Cr C. Lalescu, and M. Wilczek. "*Nature communications* 10.1 (2019): 1-8.

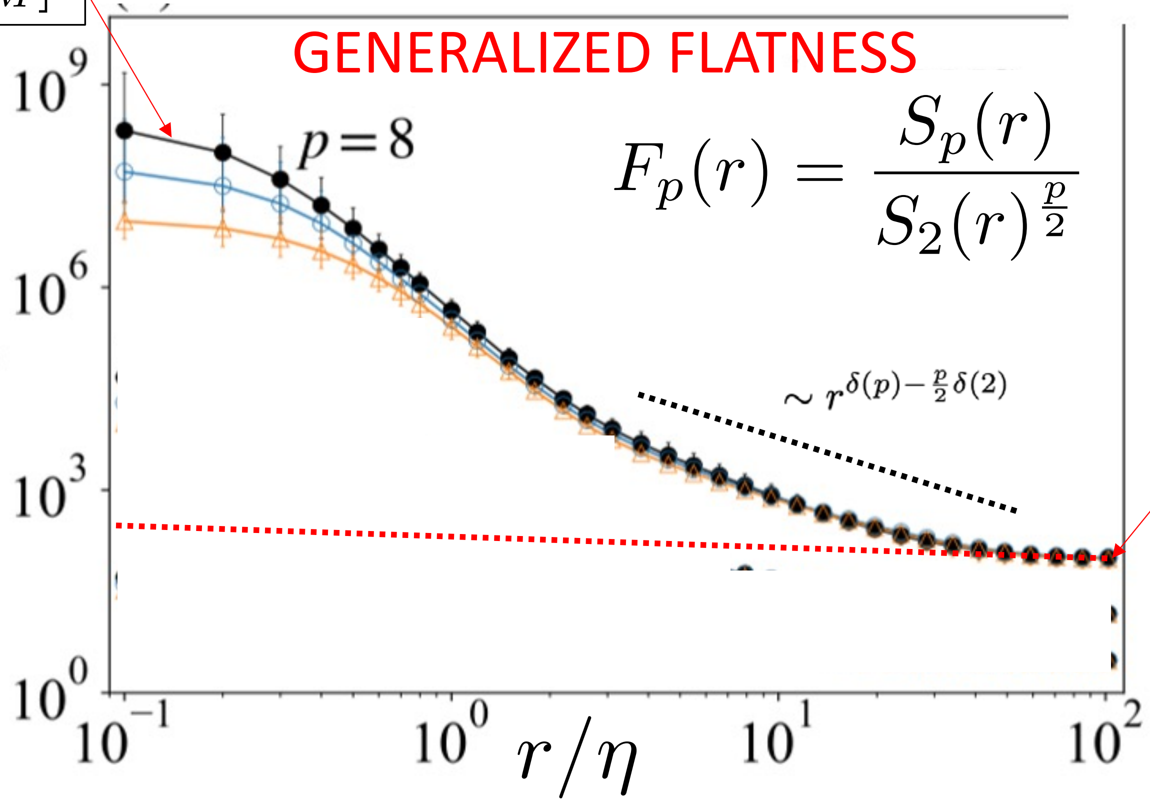
$$S_p(r) = \langle [v(x+r) - v(x)]^p \rangle \sim \epsilon^{\frac{p}{3}} r^{\zeta(p)}$$



$$\zeta(p) = \frac{p}{3} + \delta(p)$$

MULTIFRACTALS

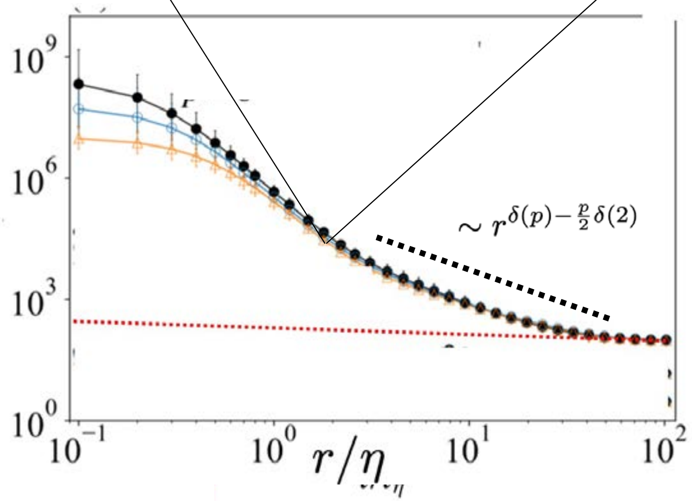
$h(x) \in [h_m, h_M]$

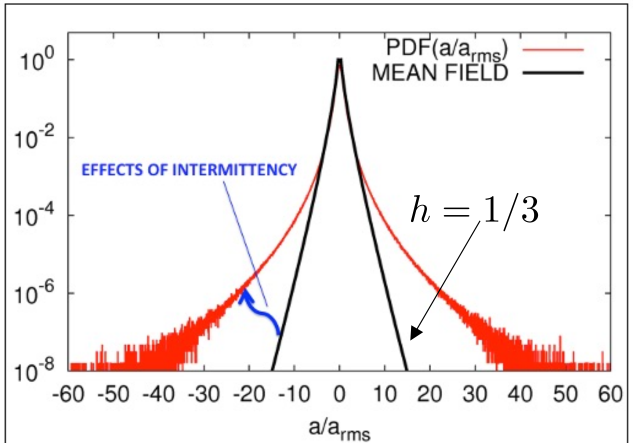


GAUSSIAN

h ~ 1/3

$$\zeta(4,\tau) = \frac{d \log S^{(4)}(\tau)}{d \log S^{(2)}(\tau)}$$



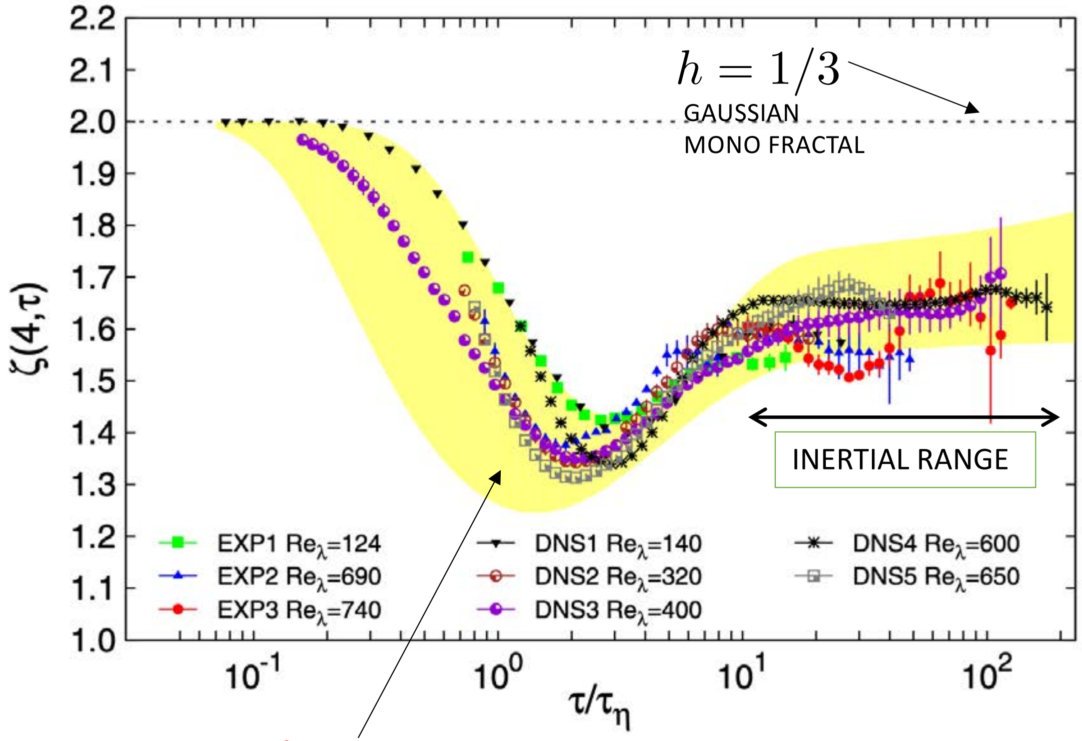


PRL 100, 254504 (2008) PHYSICAL REVIEW LETTERS week ending 27 JUNE 2008

Universal Intermittent Properties of Particle Trajectories in Highly Turbulent Flows

A. Arnèodo,[1] R. Benzi,[2] J. Berg,[3] L. Biferale,[4,*] E. Bodenschatz,[5] A. Busse,[6] E. Calzavarini,[7] B. Castaing,[1] M. Cencini,[8,*] L. Chevillard,[1] R. T. Fisher,[9] R. Grauer,[10] H. Homann,[10] D. Lamb,[9] A. S. Lanotte,[11,*] E. Lévèque,[1] B. Lüthi,[12] J. Mann,[3] N. Mordant,[13] W.-C. Müller,[6] S. Ott,[3] N. T. Ouellette,[14] J.-F. Pinton,[1] S. B. Pope,[15] S. G. Roux,[1] F. Toschi,[16,17,*] H. Xu,[5] and P. K. Yeung[18]

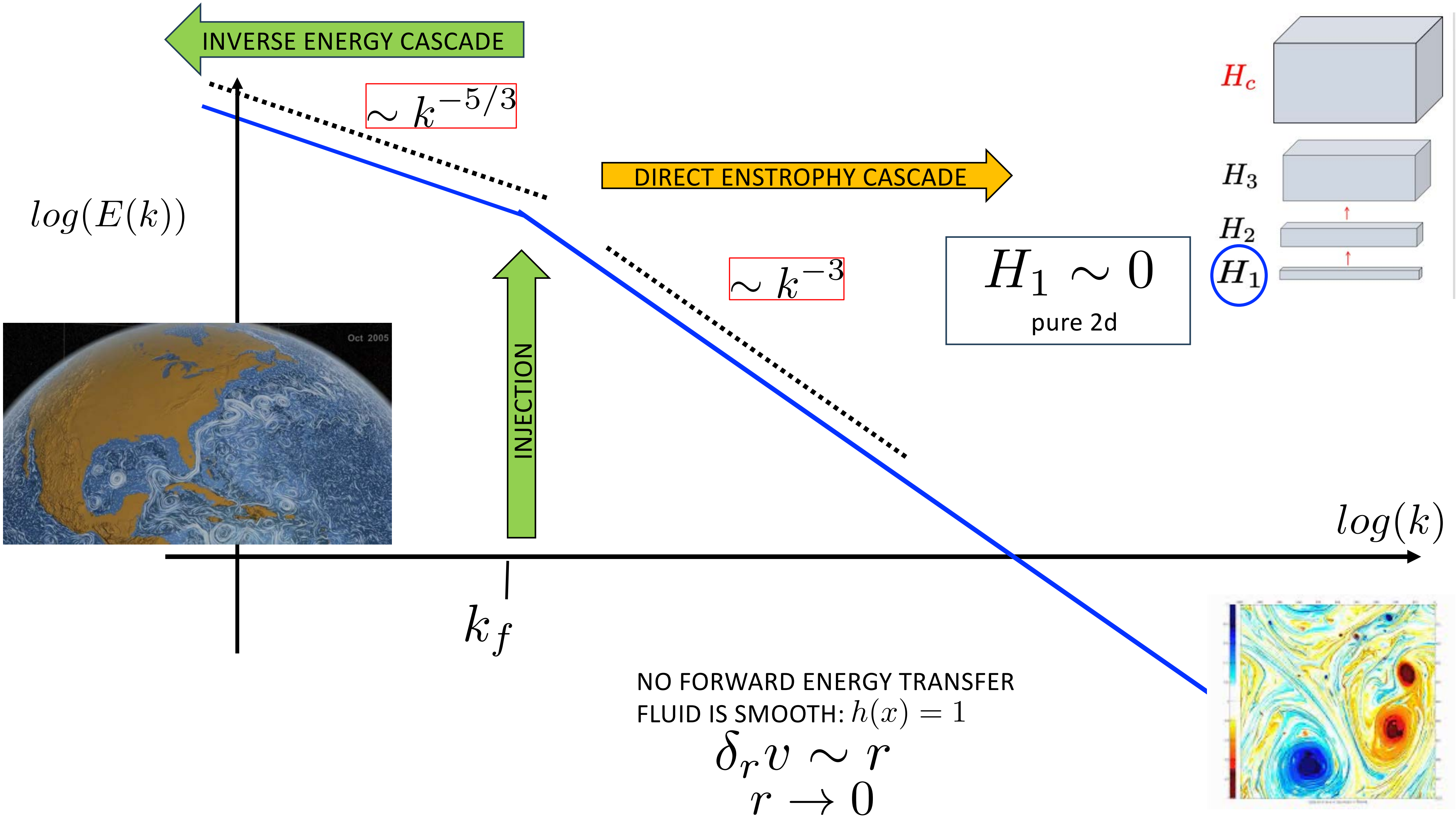
(International Collaboration for Turbulence Research)



$$\frac{1}{Re}\hat{\Delta}\hat{v}$$

TURBULENCE IS:

- STRONGLY NON-LINEAR (NON PERTURBATIVE)
- MULTI-SCALE (TRILLION OF HYDRODYNAMICAL DEGREES OF DREEDOM)
- STRONGLY OUT-OF-EQUILIBIRUM (DISSIPATIVE ANOMALY)
- NON-GAUSSIAN (INTERMITTENCY AND ANOMALOUS SCALING)
- IMPORTANT FOR MANY REAL WORLD APPLICATIONS


INVERSE ENERGY CASCADE
$\sim k^{-5/3}$
DIRECT ENSTROPHY CASCADE
$\sim k^{-3}$
$log(E(k))$
INJECTION
$log(k)$
k_f
Oct 2005
H_c
H_3
H_2
H_1
$H_1 \sim 0$
pure 2d
NO FORWARD ENERGY TRANSFER
FLUID IS SMOOTH: $h(x) = 1$
$\delta_r v \sim r$
$r \to 0$

TURBULENCE OR TURBULENCES? THIN/THICK LAYERS

MASS X ACCELERATION = INTERNAL FORCES + EXTERNAL FORCES

$$\rho[\partial_t v + v \cdot \partial v] = -\partial p + \nu \Delta v + g\theta + F(B,B) + 2\Omega \times v + F$$

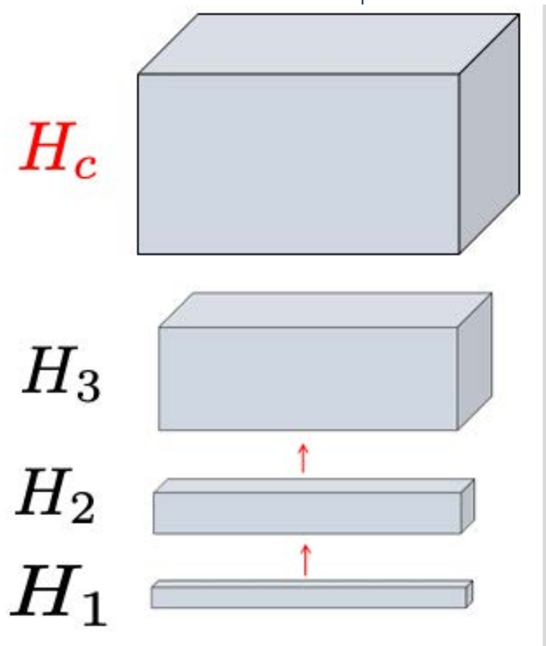
EULERIAN

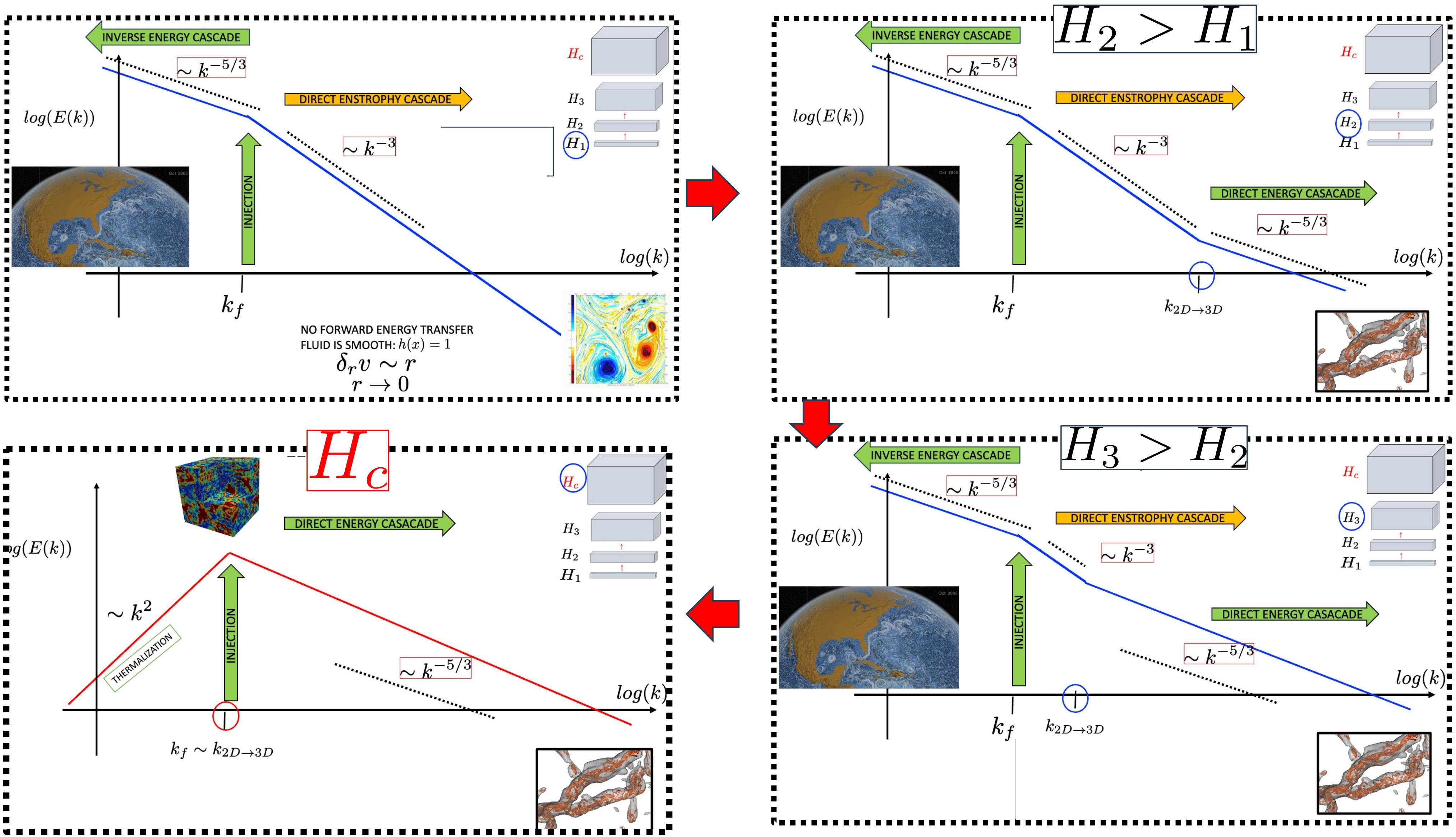
$$\partial_t \theta + v \cdot \partial \theta = \chi \partial^2 \theta$$

$$\partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B$$

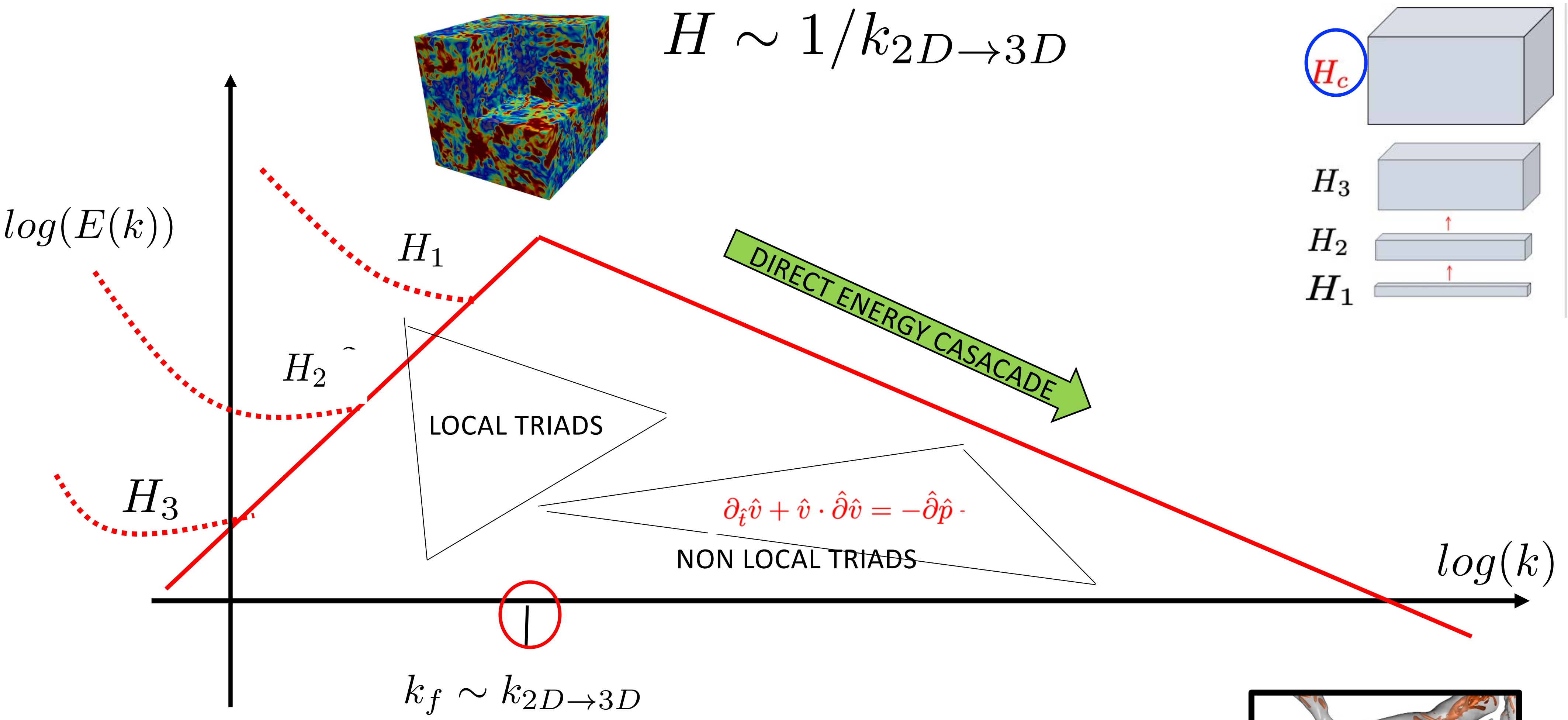
$$\partial \cdot v = 0$$

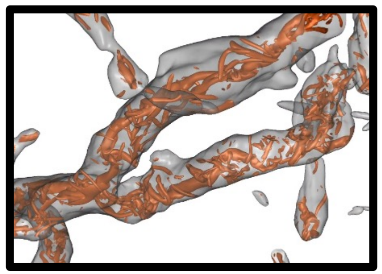
+ BOUNDARY CONDITIONS: (2D, 3D, THIN/THICK LAYERS ETC...)





INVERSE ENERGY CASCADE
$\sim k^{-5/3}$
DIRECT ENSTROPHY CASCADE
$log(E(k))$
$\sim k^{-3}$
H_c
H_3
H_2
H_1
INJECTION
$log(k)$
k_f
NO FORWARD ENERGY TRANSFER
FLUID IS SMOOTH: $h(x) = 1$
$\delta_r v \sim r$
$r \to 0$
$H_2 > H_1$
DIRECT ENERGY CASACADE
$k_{2D \to 3D}$
$H_3 > H_2$
H_c
DIRECT ENERGY CASCADE
$\sim k^2$
THERMALIZATION
$k_f \sim k_{2D \to 3D}$



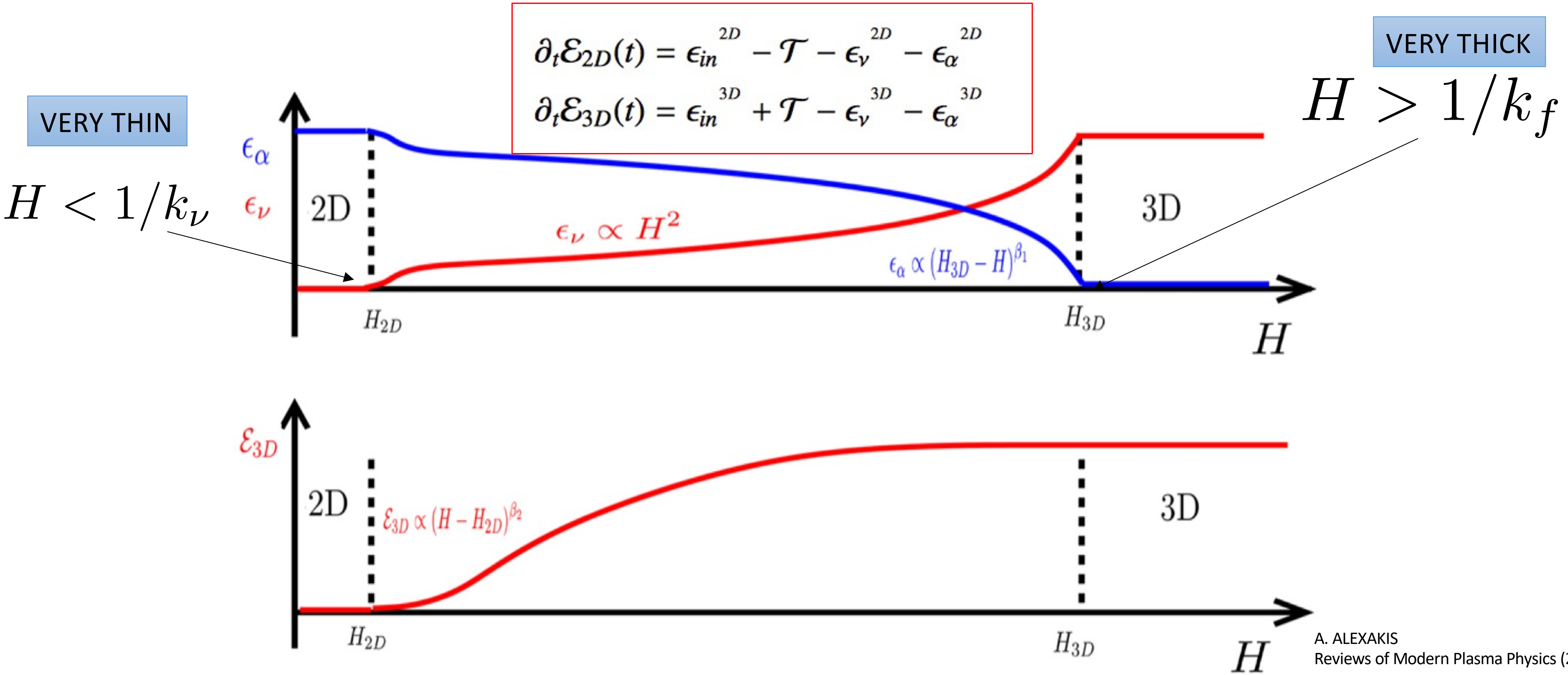


Celani, S. Musacchio, D. Vincenzi, Turbulence in more than two and less than three dimensions. Phys. Rev. Lett. 104(18), 184506 (2010)

A. Alexakis, L. B. , Cascades and transitions in turbulent flows. Phys. Rep. 767, 1–101 (2018)

DIRECT FLUX → ϵ_ν ; INVERSE FLUX → ϵ_α

$$\mathcal{I}_{\mathcal{E}} = \epsilon_\nu + \epsilon_\alpha$$



A. ALEXAKIS
Reviews of Modern Plasma Physics (2023)

TURBULENCE OR TURBULENCES? ROTATION

MASS X ACCELERATION = INTERNAL FORCES + EXTERNAL FORCES

$$\rho[\partial_t v + v \cdot \partial v] = -\partial p + \nu \Delta v + g\theta + F(B,B) + 2\Omega \times v + F$$

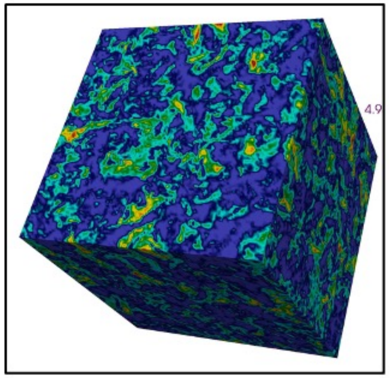
EULERIAN

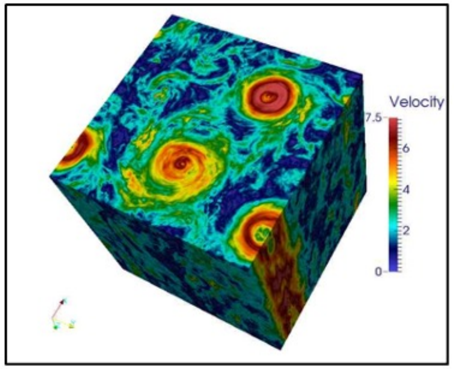
$$\partial_t \theta + v \cdot \partial \theta = \chi \partial^2 \theta$$

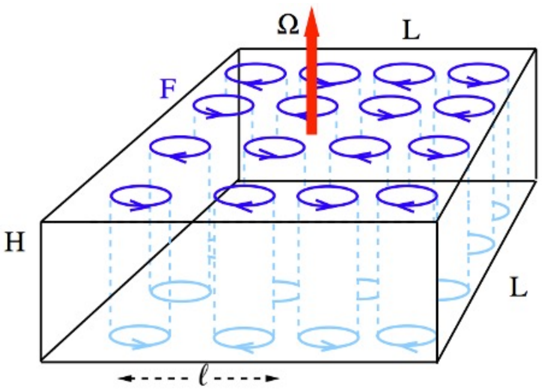
$$\partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B$$

$$\partial \cdot v = 0$$

+ BOUNDARY CONDITIONS: (2D, 3D, THIN/THICK LAYERS ETC...)



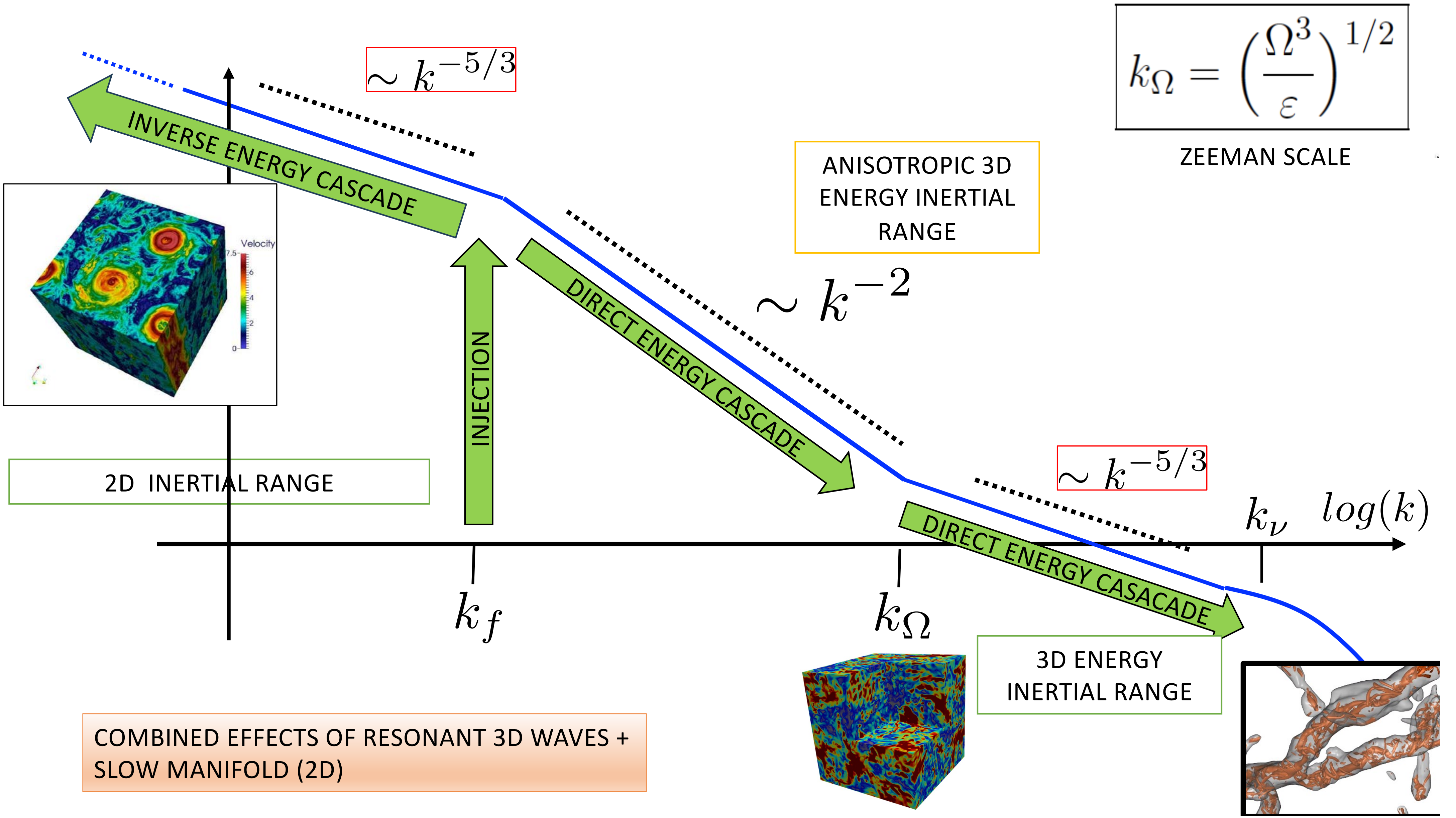




NEW CONTROL PARAMETER

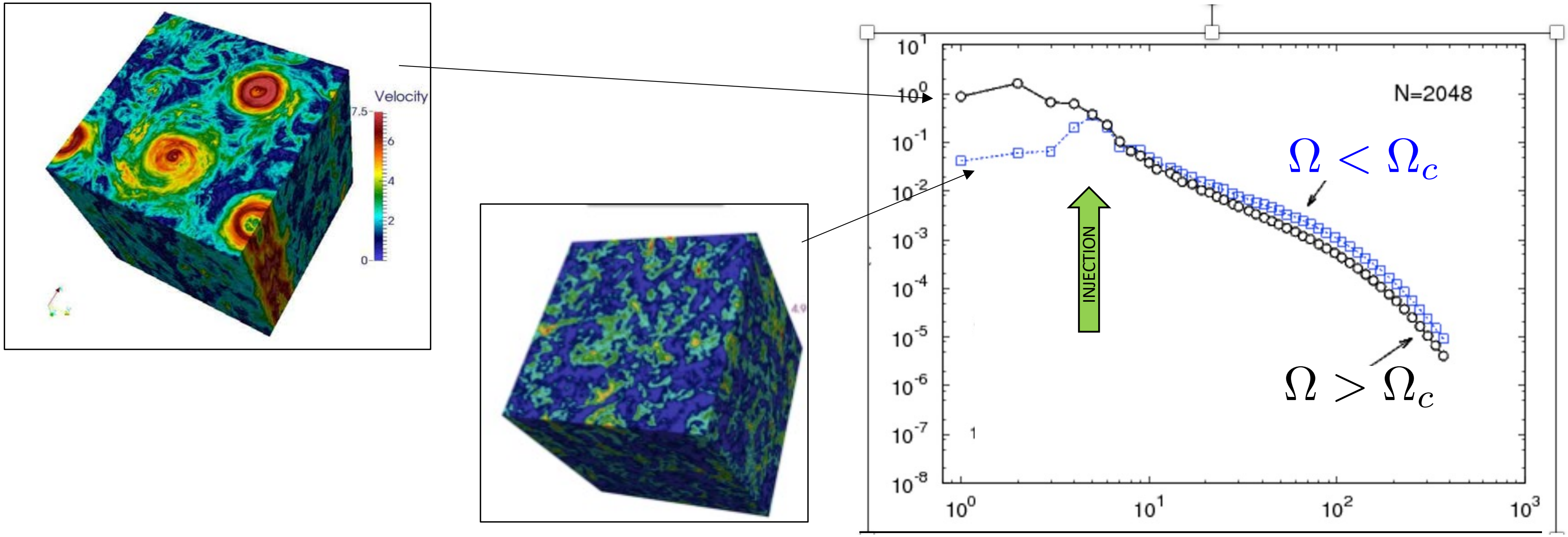
ROOSBY NUMBER

$$Ro = \frac{U}{2\Omega \ell_{in}}$$

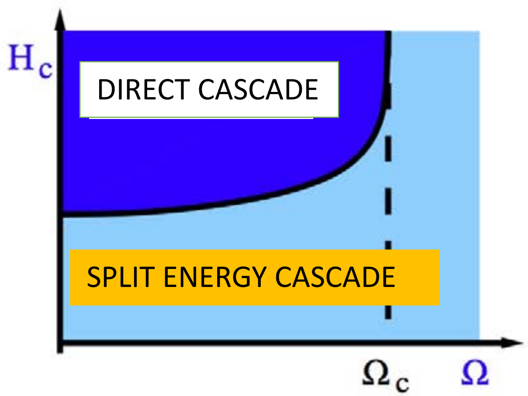


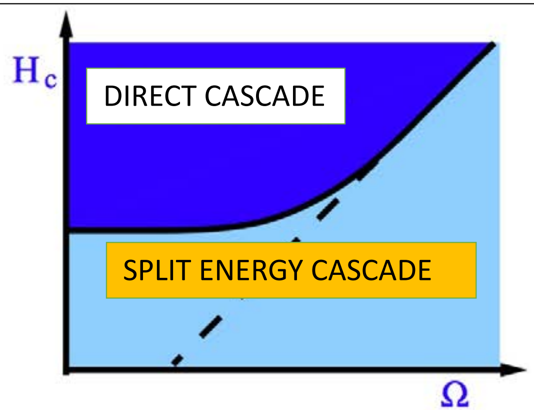
$k_\Omega = \left(\frac{\Omega^3}{\varepsilon}\right)^{1/2}$
ZEEMAN SCALE
$\sim k^{-5/3}$
INVERSE ENERGY CASCADE
Velocity
ANISOTROPIC 3D ENERGY INERTIAL RANGE
$\sim k^{-2}$
INJECTION
DIRECT ENERGY CASCADE
$\sim k^{-5/3}$
2D INERTIAL RANGE
k_ν
$log(k)$
DIRECT ENERGY CASACADE
k_f
k_Ω
3D ENERGY INERTIAL RANGE
COMBINED EFFECTS OF RESONANT 3D WAVES + SLOW MANIFOLD (2D)

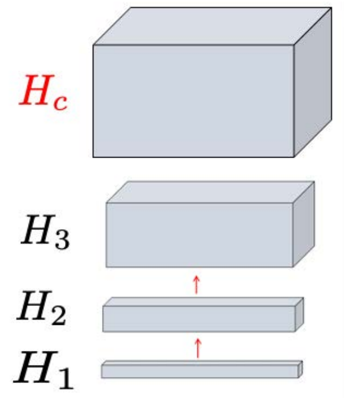
DOES THE INVERSE CASCADE PERSIST FOR ANY ASPECT RATIO? AND ROTATION STRONG ENOUGH?



A.Sen et al Jour Atmos Science 68, 2757 (2011)
E. Yarom et al PoF 25, 085105 (2013)
A. Campagne et al PoF 26, 125112 (2014)
L.B. F. Bonaccorso et al PRX 6, 041036 (2016)
E. Deusebio et al PRE 90, 023005 (2014)
A. Alexakis JFM 769, 46 (2015)







TURBULENCE OR TURBULENCES? STRATIFICATION

MASS X ACCELERATION = INTERNAL FORCES + EXTERNAL FORCES

EULERIAN

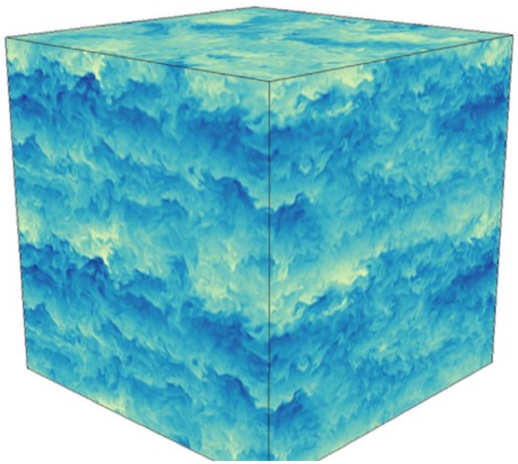
$$\rho[\partial_t v + v \cdot \partial v] = -\partial p + \nu \Delta v + g\theta + F(B,B) + 2\Omega \times v + F$$

$$\partial_t \theta + v \cdot \partial \theta = \chi \partial^2 \theta$$

$$\partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B$$

$$\partial \cdot v = 0$$

+ BOUNDARY CONDITIONS: (2D, 3D, THIN/THICK LAYERS ETC...)



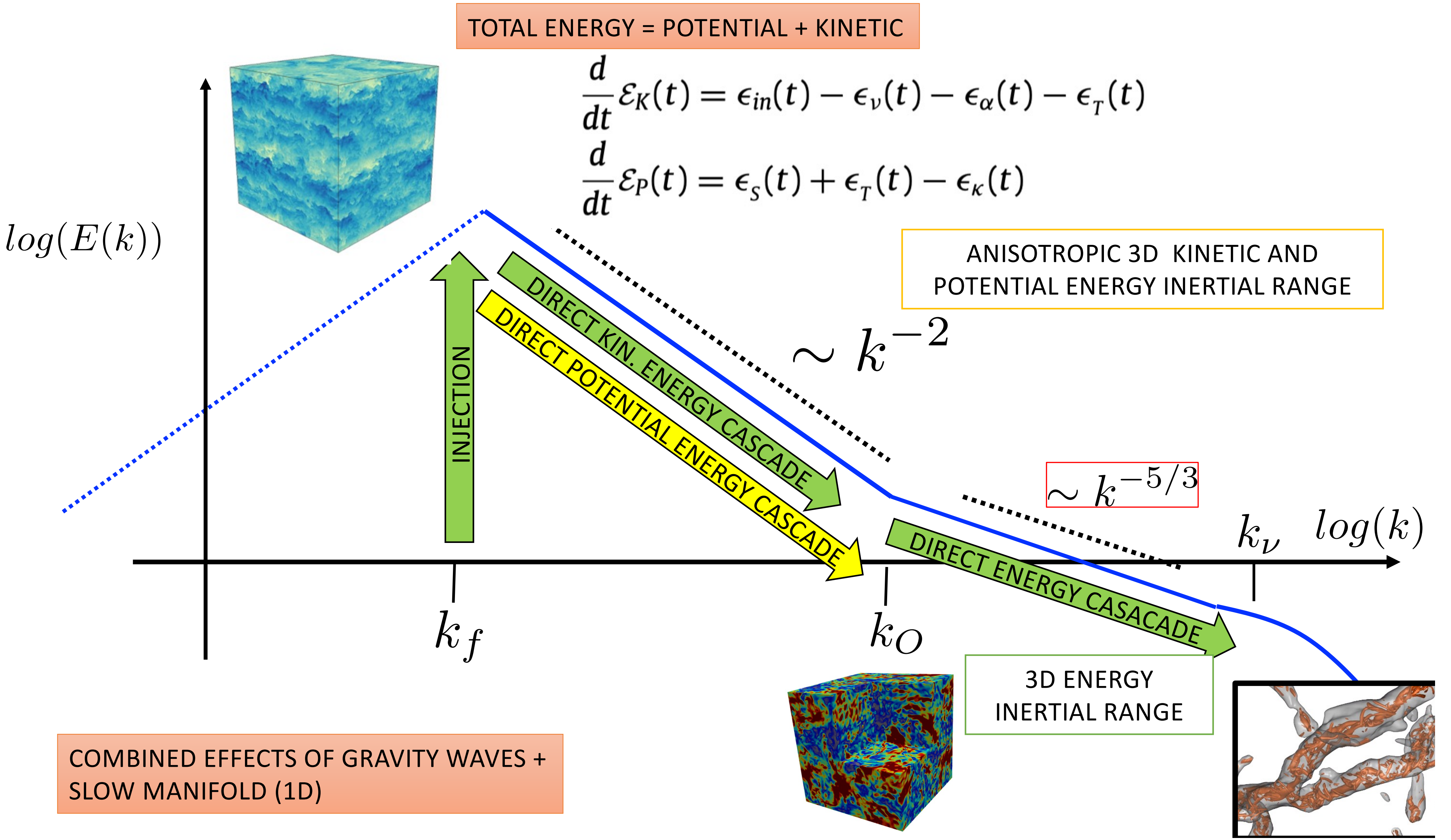
NEW CONTROL PARAMETER

BRUNT-VAISALA FREQUENCY -> FROUDE NUMBER

σ = MEAN STRATIFICATION

$$N = \sqrt{g\sigma}$$

$$Fr = \frac{U}{N\ell_{in}}$$



TOTAL ENERGY = POTENTIAL + KINETIC
$\frac{d}{dt}\mathcal{E}_K(t) = \epsilon_{in}(t) - \epsilon_\nu(t) - \epsilon_\alpha(t) - \epsilon_T(t)$
$\frac{d}{dt}\mathcal{E}_P(t) = \epsilon_S(t) + \epsilon_T(t) - \epsilon_\kappa(t)$
$log(E(k))$
ANISOTROPIC 3D KINETIC AND POTENTIAL ENERGY INERTIAL RANGE
$\sim k^{-2}$
INJECTION
DIRECT KIN. ENERGY CASCADE
DIRECT POTENTIAL ENERGY CASCADE
$\sim k^{-5/3}$
k_ν
$log(k)$
DIRECT ENERGY CASACADE
k_f
k_O
3D ENERGY INERTIAL RANGE
COMBINED EFFECTS OF GRAVITY WAVES + SLOW MANIFOLD (1D)

DOES THE INVERSE CASCADE PERSIST FOR ANY ASPECT RATIO? AND STRATIFICATION STRONG ENOUGH?

MASS X ACCELERATION = INTERNAL FORCES + EXTERNAL FORCES

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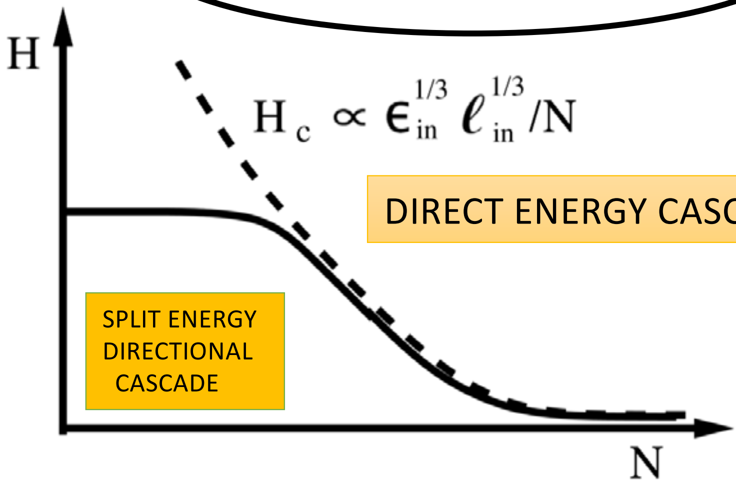
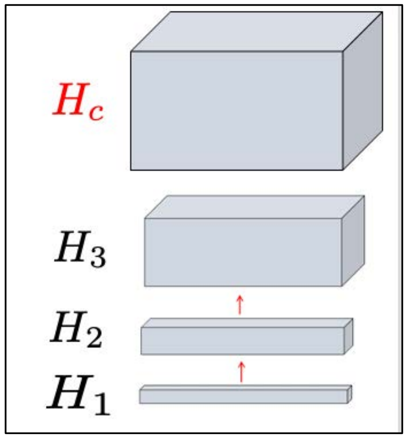
$$\partial_t \theta + v \cdot \partial \theta = \chi \partial^2 \theta$$

$$\partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B$$

$$\partial \cdot v = 0$$

+ BOUNDARY CONDITIONS: (2D, 3D, THIN/THICK LAYERS ETC...)





Sozza, G. Boffetta, P. Muratore-Ginanneschi et al., Dimensional transition of energy cascades in stably stratified forced thin fluid layers. Phys. Fluids 27(3), 035112 (2015)

TURBULENCE OR TURBULENCES? ROTATION+STRATIFICATION

MASS X ACCELERATION = INTERNAL FORCES + EXTERNAL FORCES

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$$\rho[\partial_t v + v \cdot \partial v] = -\partial p + \nu \Delta v + g\theta + F(B,B) + 2\Omega \times v + F$$

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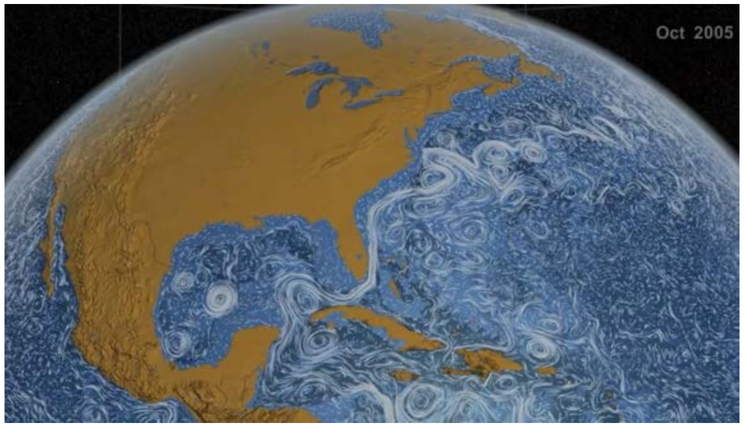
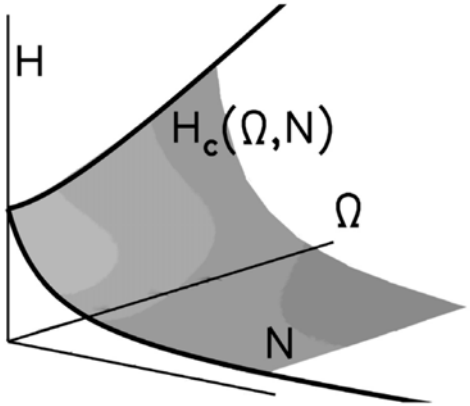
$$\partial_t B + v \cdot \partial B = B \cdot \partial v + \chi \partial^2 B$$

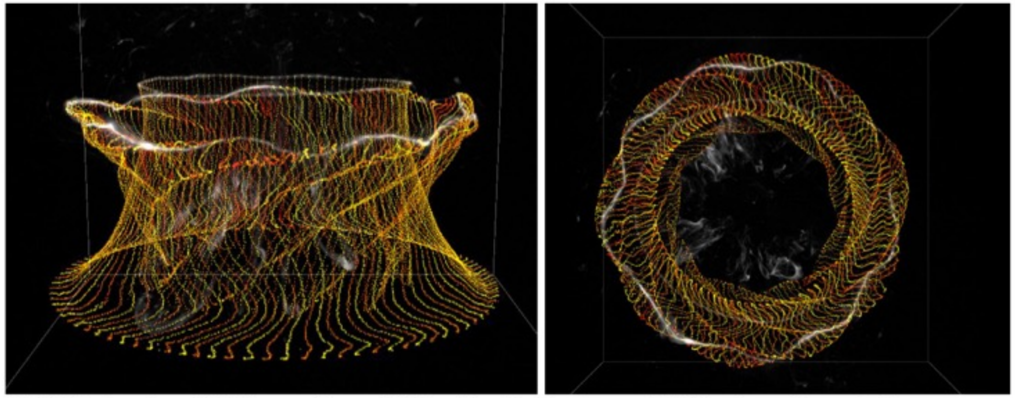
$$\partial \cdot v = 0$$

+ BOUNDARY CONDITIONS: (2D, 3D, THIN/THICK LAYERS ETC...)





HELICITY INJECTION



M.W. Scheeler, W.M. van Rees, H. Kedia, D. Kleckner, W.T. Irvine,

INVERSE ENERGY CASCADE IN 3D!

$$\begin{cases} E = \sum_{\boldsymbol{k}} |u^+(\boldsymbol{k})|^2 + |u^-(\boldsymbol{k})|^2; \\ H = \sum_{\boldsymbol{k}} k(|u^+(\boldsymbol{k})|^2 - |u^-(\boldsymbol{k})|^2). \end{cases}$$

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HOME > SCIENCE > VOL. 357, NO. 6350 > COMPLETE MEASUREMENT OF HELICITY AND ITS DYNAMICS IN VORTEX TUBES

Complete measurement of helicity and its dynamics in vortex tubes

MARTIN W. SCHEELER, WIM M. VAN REES, HRIDESH KEDIA, DUSTIN KLECKNER, AND WILLIAM T. M. IRVINE Authors Info & Affiliations

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ODD VISCOSITY

Pattern formation by non-dissipative arrest of turbulent cascades

Xander M. de Wit,[1] Michel Fruchart,[2,3] Tali Khain,[3] Federico Toschi,[1,4] and Vincenzo Vitelli[3,5]

FLUX-LOOP CASCADE: ZERO-FLUX & OUT-OF-EQUILIBRIUM

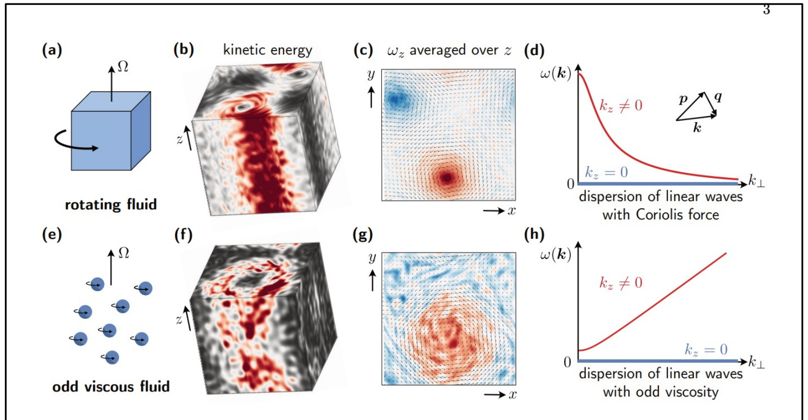
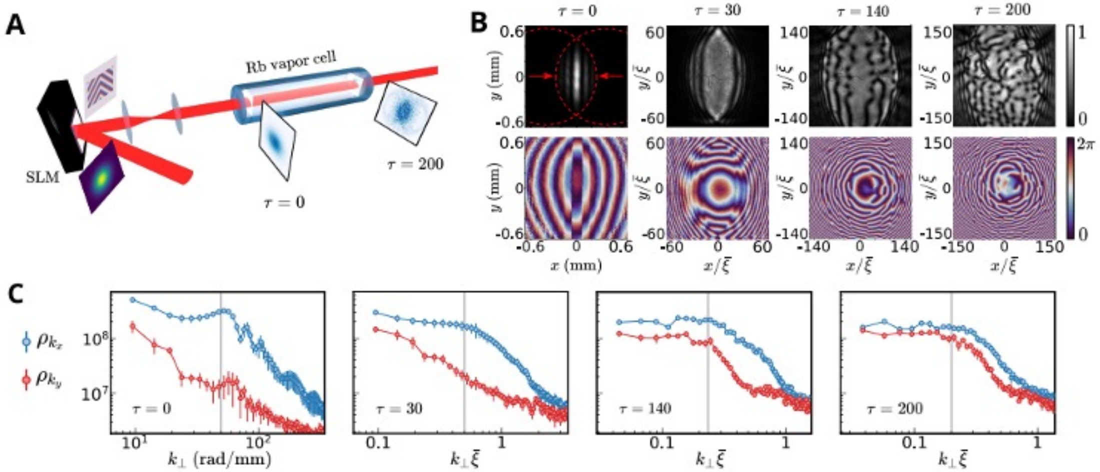


FIG. 2. **Two-dimensionalization in rotating and odd turbulence.** We compare (a-d) a turbulent fluid flow rotating

Inverse energy cascade in two-dimensional quantum turbulence in a fluid of light

Myrann Abobaker, Wei Liu, Tangui Aladjidi, Alberto Bramati, and Quentin Glorieux*



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HOME > SCIENCE > VOL. 364, NO. 6447 > GIANT VORTEX CLUSTERS IN A TWO-DIMENSIONAL QUANTUM FLUID

REPORT

Giant vortex clusters in a two-dimensional quantum fluid

GUILLAUME GAUTHIER, MATTHEW T. REEVES, XIAOQUAN YU, ASHTON S. BRADLEY, MARK A. BAKER, THOMAS A. BELL, HALINA RUBINSZTEIN-DUNLOP, MATTHEW J. DAVIS, AND TYLER W. NEELY Authors Info & Affiliations

SCIENCE • 28 Jun 2019 • Vol 364, Issue 6447 • pp. 1264-1267 • DOI: 10.1126/science.aat5718

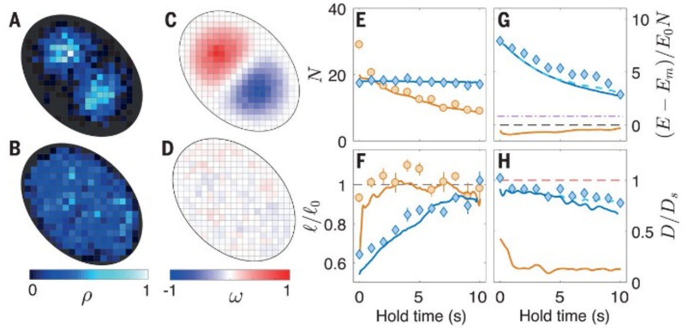


Fig. 3 Evidence of vortex cluster metastability.

TURBULENCE IS

HIGH COMPLEXITY:

- STRONGLY NON-LINEAR (NON PERTURBATIVE)
- MULTI-SCALE (BILLIONS OF HYDRODYNAMICAL DEGREES OF DREEDOM)
- STRONGLY OUT-OF-EQUILIBIRUM (DISSIPATIVE ANOMALY IN 3D)
- NON-GAUSSIAN (INTERMITTENCY AND ANOMALOUS SCALING)

RICH PHENOMENOLOGY:

-DIRECT ENERGY CASCADE - 3D
-INVERSE ENERGY CASCADE + DIRECT ENSTROPHY CASCADE - 2D
-FORWARD AND INVERSE ENERGY CASCADE – QUASI 2D, QUASI 3D, ROTATION, STRATIFICATION
-INVERSE ENERGY CASCADE - 3D + HELICITY
-FLUX-LOOP (0 FLUX -> BIDIRECTIONAL CASCADE) ODD-VISCOSITY FLUIDS, STRATIFICATION 2D

MULTI FEATURES:

A STORY OF: LOCAL VS NON-LOCAL FOURIER INTERACTIONS, INERTIAL AND GRAVITY WAVES, SLOW-MANIFOLDS, TOPOLOGICAL CONSTRAINTS

J. von NEUMANN (1949)

These considerations justify the view that a considerable mathematical effort towards a detailed understanding of the mechanism of turbulence is called for. The entire experience with the subject indicates that the purely analytical approach is beset with difficulties, which at this moment are still prohibitive. The reason for this is probably as was indicated above: That our intuitive relationship to the subject is still too loose — not having succeeded at anything like deep mathematical penetration in any part of the subject, we are still quite disoriented as to the relevant factors, and as to the proper analytical machinery to be used.

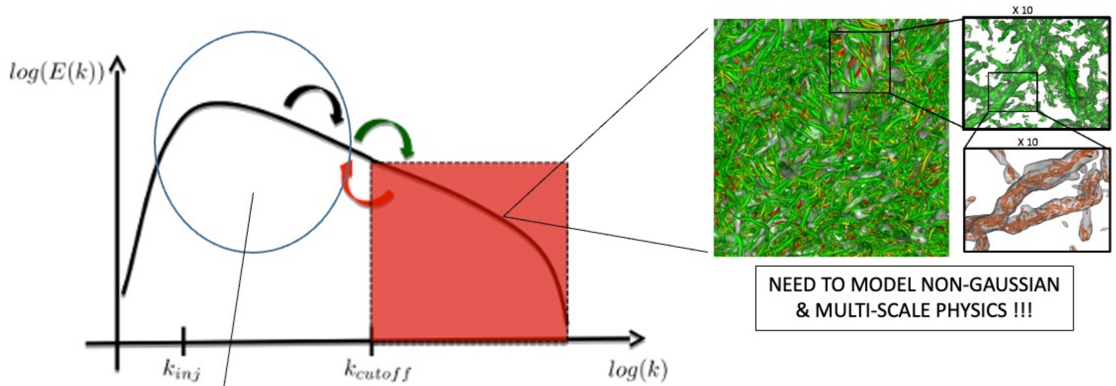
Under these conditions there might be some hope to 'break the deadlock' by extensive, but well-planned, computational efforts. It must be admitted that the problems in question are too vast to be solved by a direct computational attack, that is, by an outright calculation of a representative family of special cases. There are, however, strong indications that one could name certain strategic points in this complex, where relevant information must be obtained by direct calculations. If this is properly done, and the operation is then repeated on the basis of broader information then becoming available, etc., there is a reasonable chance of effecting real penetrations in this complex of problems and gradually developing a useful, intuitive relationship to it. This should, in the end, make an attack with analytical methods, that is truly more mathematical, possible.[1]

HOW/WHERE TO USE DATA-DRIVEN TOOLS TO ATTACK TURBULENCE?

1. TO SUPER-RESOLVE/MODEL SMALL-SCALES PROPERTIES IN DNS MODELS:

$$\partial_t v + \nabla(v \otimes v) = -\nabla p + \nu \Delta v$$

$$\overline{\boldsymbol{v}}(\boldsymbol{x},t) \equiv \int_\Omega d\boldsymbol{y}\ G(|\boldsymbol{x}-\boldsymbol{y}|)\ \boldsymbol{v}(\boldsymbol{y},t) = \sum_{\boldsymbol{k}\in\mathbb{Z}^3} G(\boldsymbol{k})\ \hat{\boldsymbol{v}}(\boldsymbol{k},t)e^{i\boldsymbol{k}\boldsymbol{x}}$$



$$\partial_t \overline{\boldsymbol{v}} + \nabla \cdot (\overline{\boldsymbol{v}} \otimes \overline{\boldsymbol{v}}) = -\nabla \overline{p} + \nabla \cdot \tau^{\Delta}(\boldsymbol{v},\boldsymbol{v}) + \nu \Delta \overline{\boldsymbol{v}}$$

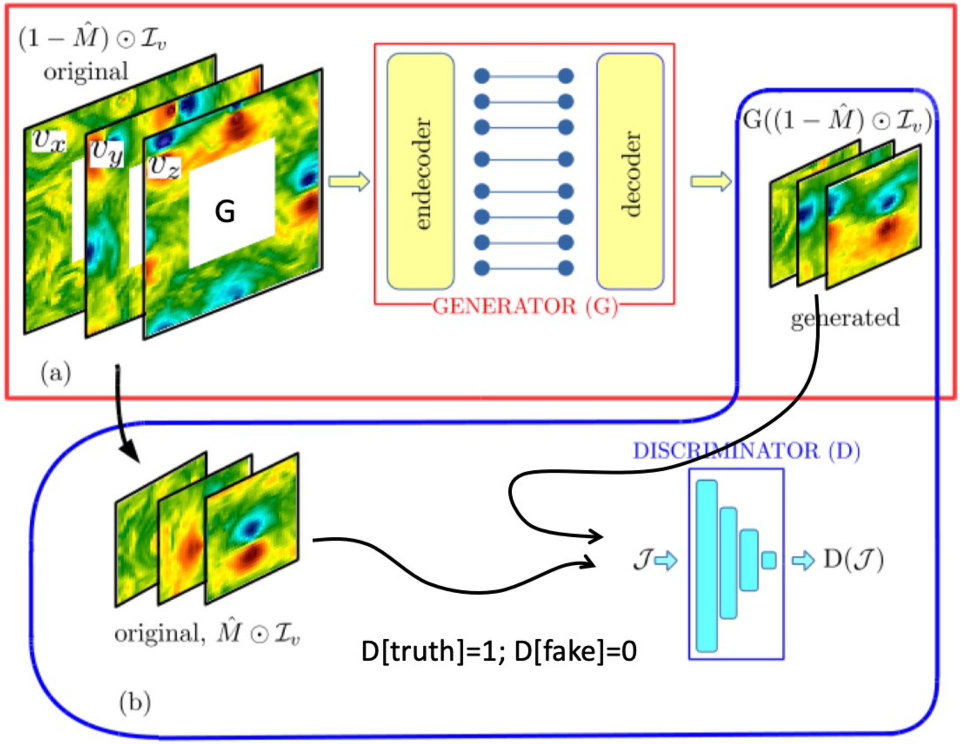
$$\tau_{ij}^{\Delta}(\boldsymbol{v},\boldsymbol{v}) = \overline{v_i v_j} - \overline{v}_i \overline{v}_j \xrightarrow{??????} \tau(\bar{v},\bar{v})$$

Solver-in-the-Loop: Learning from Differentiable Physics to Interact with Iterative PDE-Solvers
Kiwon Um, Robert Brand, Yun (Raymond)Fei, Philipp Holl, Nils Thuerey

HOW/WHERE TO USE DATA-DRIVEN TOOLS TO ATTACK TURBULENCE?

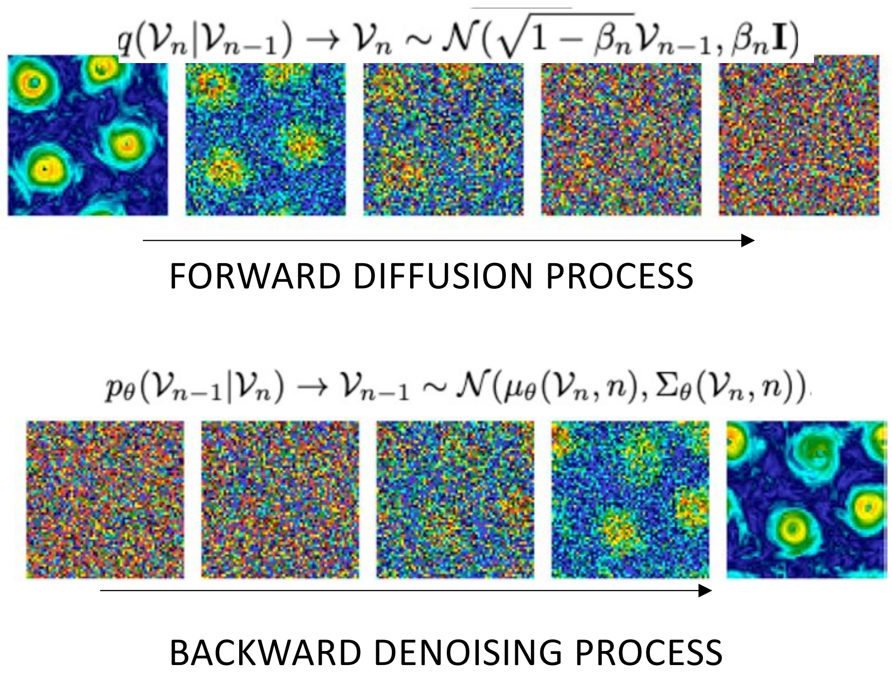
2. TO GENERATE/AUGMENT PARTIAL FIELD OBSERVATIONS/MEASUREMENT

GENERATIVE ADVERSARIAL TOOLS



Multi-scale reconstruction of turbulent rotating flows with proper orthogonal decomposition and generative adversarial networks T Li, M Buzzicotti, L B., F Bonaccorso, S Chen, M Wan Journal of Fluid Mechanics 971, A3 (2023)

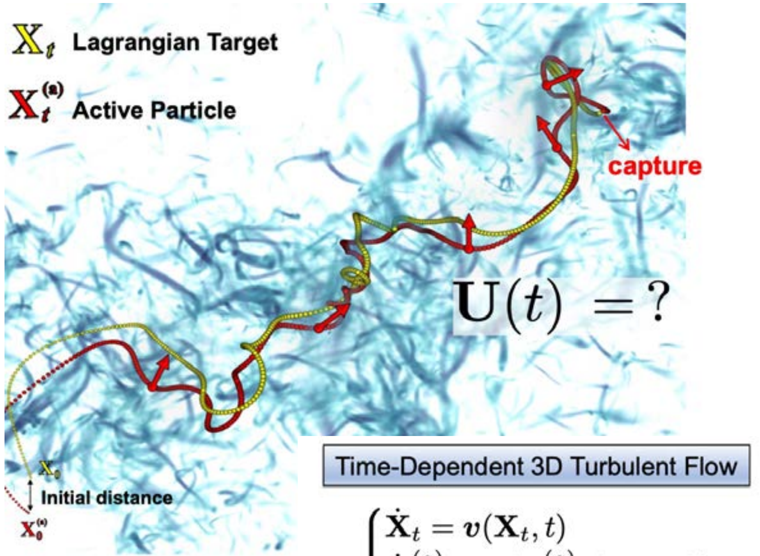
GENERATIVE DIFFUSION MODELS



Synthetic lagrangian turbulence by generative diffusion models
T Li, L B., F Bonaccorso, MA Scarpolini, M Buzzicotti arXiv:2307.08529

HOW/WHERE TO USE DATA-DRIVEN TOOLS TO ATTACK TURBULENCE?

3. FOR OPTIMAL NAVIGATION POLICIES OF LAGRANGIAN ACTIVE SMALL OBJECTS

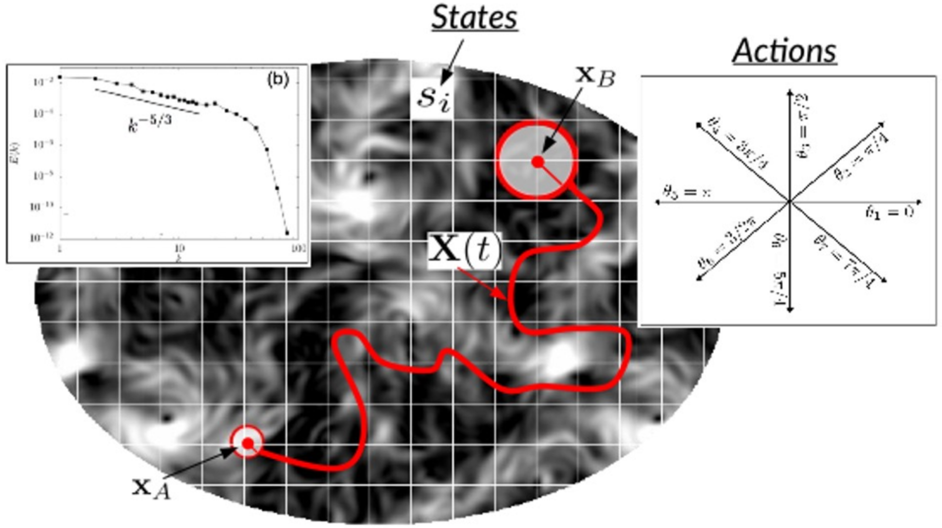


$$\begin{cases} \dot{\mathbf{X}}_t = \boldsymbol{v}(\mathbf{X}_t, t) \\ \dot{\mathbf{X}}_t^{(a)} = \boldsymbol{v}(\mathbf{X}_t^{(a)}, t) + \mathbf{U}(t) \\ \mathbf{U}(t) = V_s \hat{\boldsymbol{n}}(t) \end{cases}$$

Small active control compared to the underling flow!

Goal: **minimize** the **separation/capture** in a finite time horizon

Reinforcement Learning; *Policy Gradient Methods*



Optimal tracking strategies in a turbulent flow. Calascibetta, C., B. L., Borra, F. *et al.* *Commun Phys* **6**, 256 (2023).

Zermelo's problem: optimal point-to-point navigation in 2D turbulent flows using reinforcement learning. L B. F Bonaccorso, M Buzzicotti, P Clark Di Leoni, K Gustavsson Chaos: An Interdisciplinary Journal of Nonlinear Science 29 (2019)

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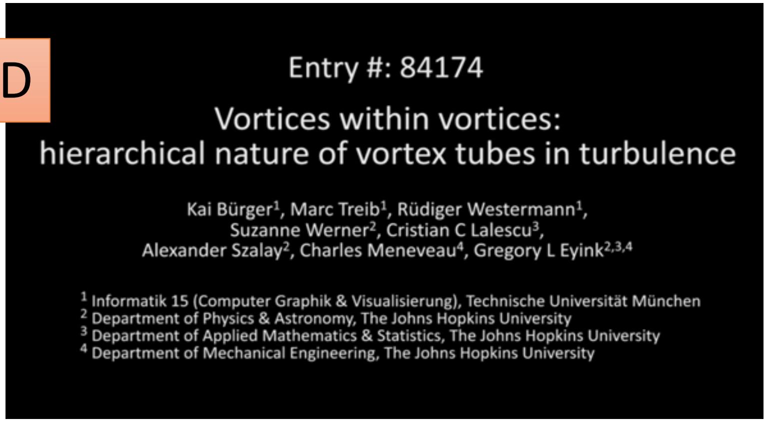
HOW TO BREAK THE DEDLOCK?

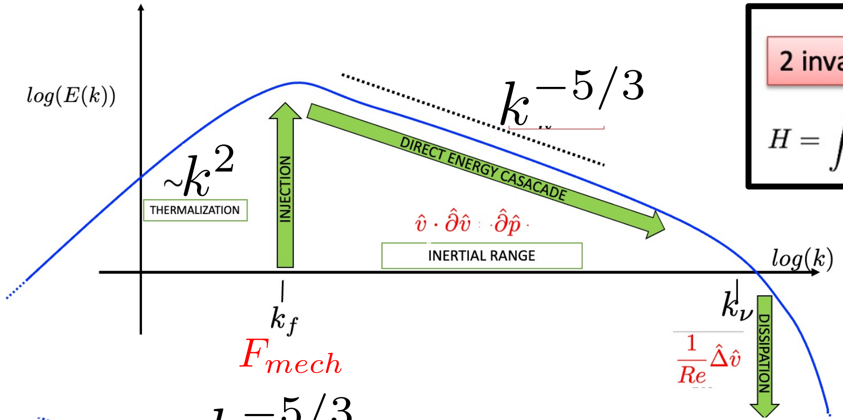
USE ANY POSSIBILE TOOL: SYNERGIES BETWEEN GOOD ANALITICAL THEORIES/MODELS + WELL PLANNED EXPERIMENTS/NUMERICAL SIMULATIONS + DATA-DRIVEN APPROACH

BASIC FUNDAMENTAL OPEN PROBLEMS ABOUT GLOBAL AND LOCAL PROPERTIES

1. WE DO NOT CONTROL THE MEAN ENERGY FLUX: DOES THE INJECTED ENERGY FLOW TOWARDS SMALL-SCALES OR TOWARDS LARGE-SCALES (OR BOTH)?
2. WE DO NOT CONTROL THE SCALE-BY-SCALE MEAN ENERGY SPECTRUM
3. WE DO NOT CONTROL NEITHER EULERIAN NOR LAGRANGIAN FLUCTUATIONS AROUND THE MEAN GLOBAL PROPERTIES

3D

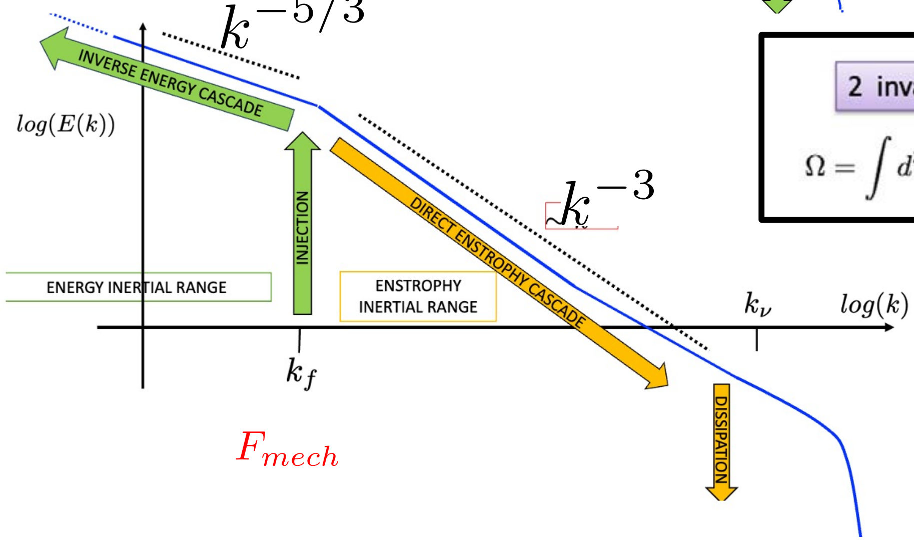




2 invariants (only one positive definite)

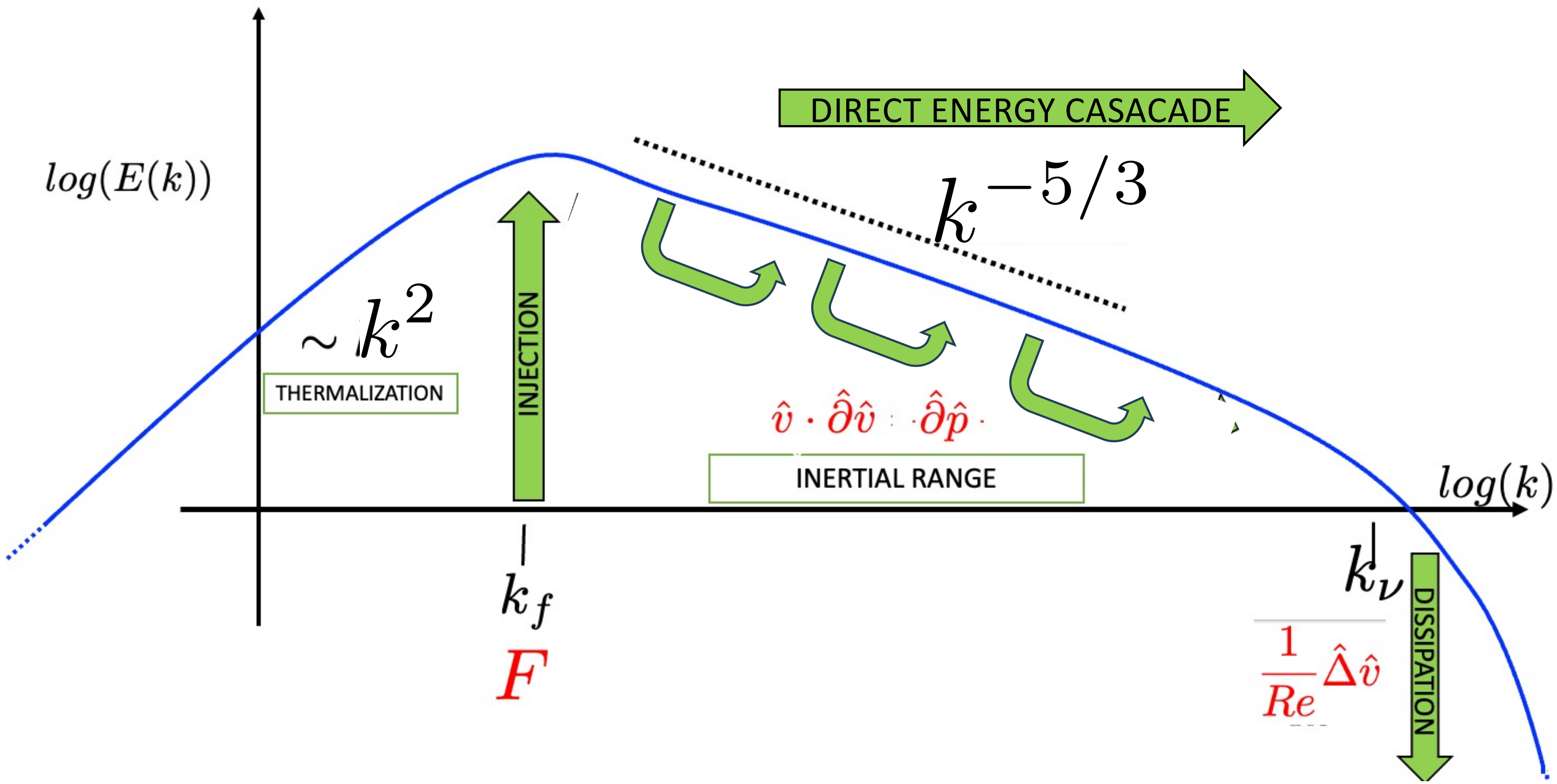
$$H = \int d^3x\, \omega \cdot \mathbf{v} \quad E = \int d^3x\, \mathbf{v} \cdot \mathbf{v}$$

2D

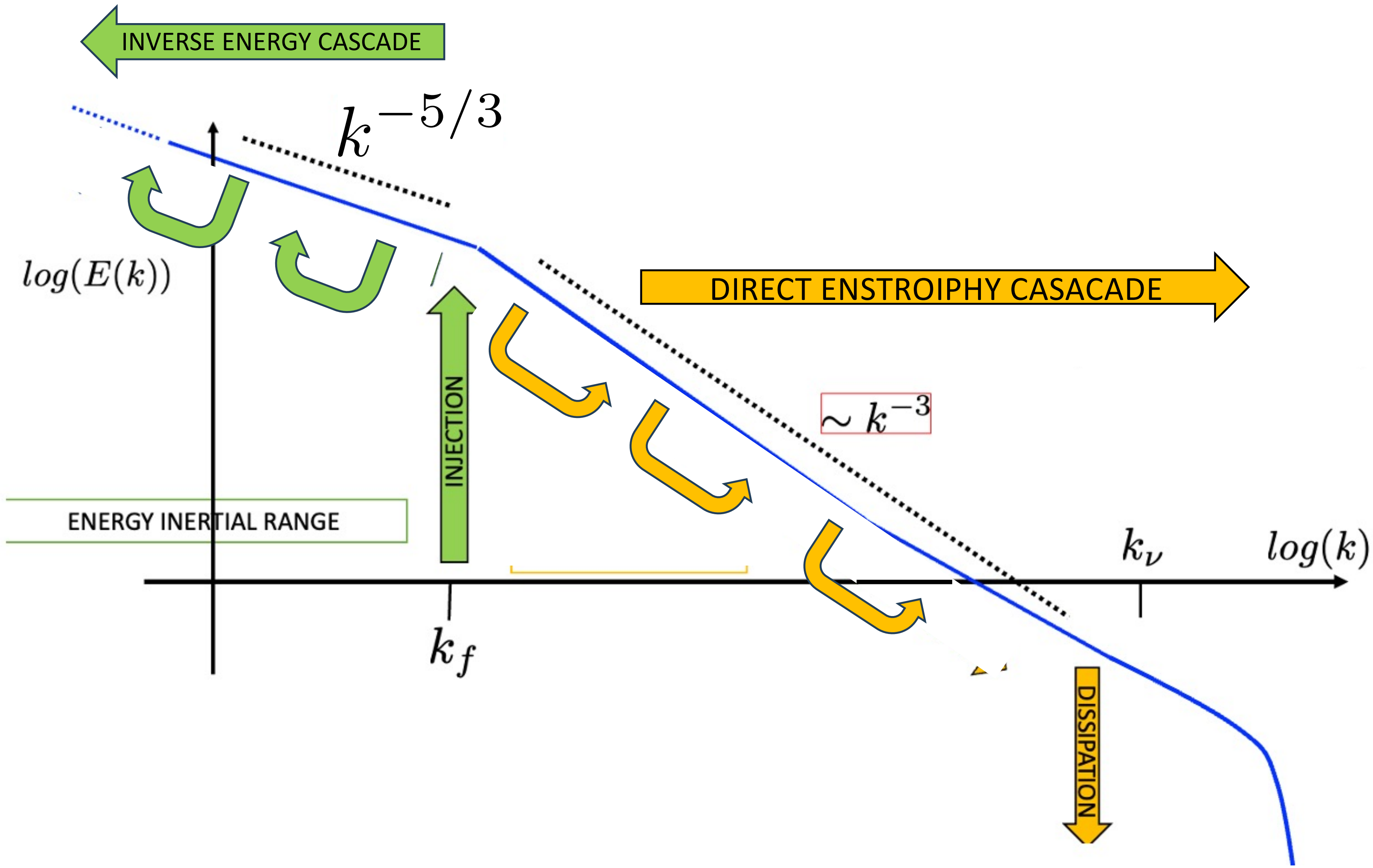


2 invariants (positive definite)

$$\Omega = \int d^3x\, \omega \cdot \omega \quad E = \int d^3x\, \mathbf{v} \cdot \mathbf{v}$$



DIRECT ENERGY CASACADE
$log(E(k))$
$k^{-5/3}$
$\sim k^2$
THERMALIZATION
INJECTION
$\hat{v} \cdot \hat{\partial}\hat{v} \quad \hat{\partial}\hat{p}$
INERTIAL RANGE
$log(k)$
k_f
F
k_ν
$\frac{1}{Re}\hat{\Delta}\hat{v}$
DISSIPATION



INVERSE ENERGY CASCADE
$k^{-5/3}$
$log(E(k))$
DIRECT ENSTROIPHY CASACADE
INJECTION
$\sim k^{-3}$
ENERGY INERTIAL RANGE
k_ν
$log(k)$
k_f
DISSIPATION

BASIC FUNDAMENTAL PROBLEMS ABOUT GLOBAL AND LOCAL PROPERTIES

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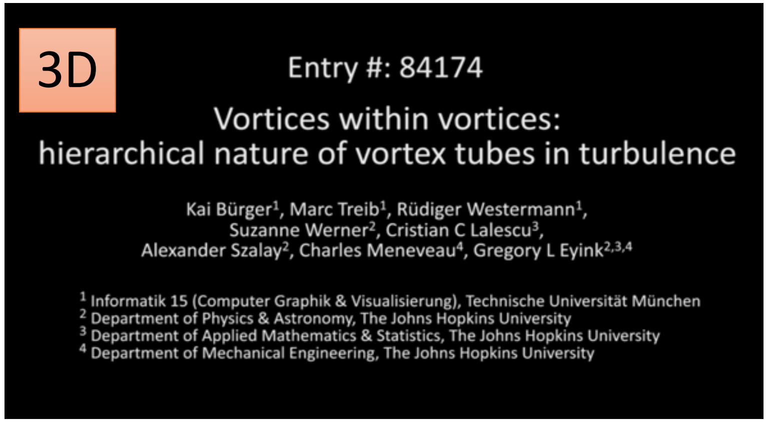

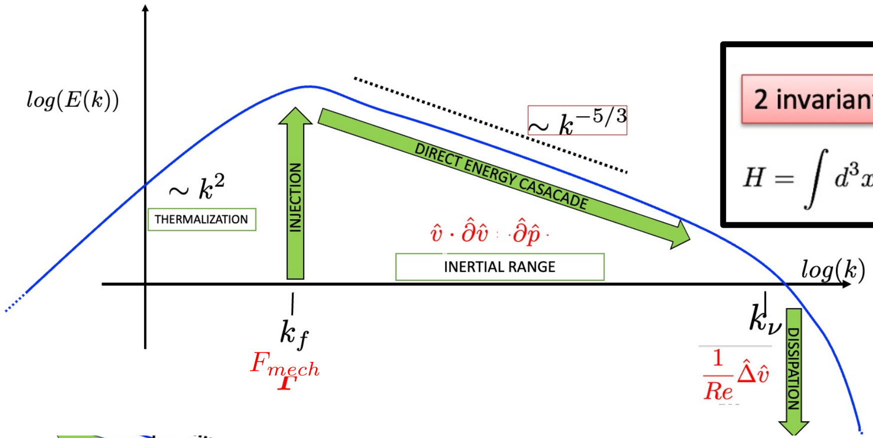

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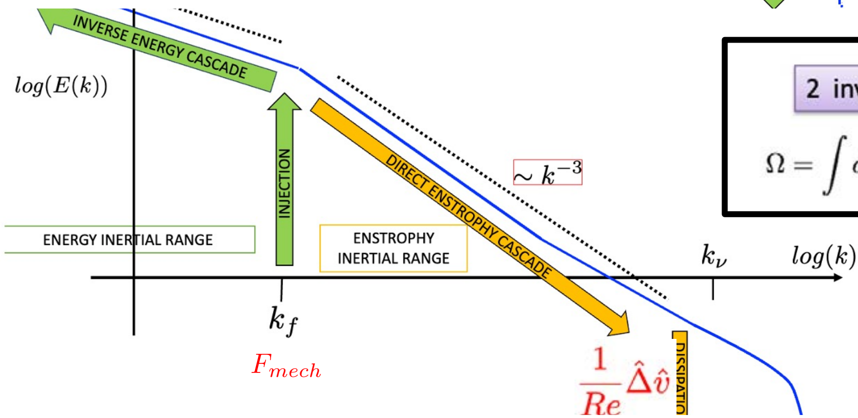
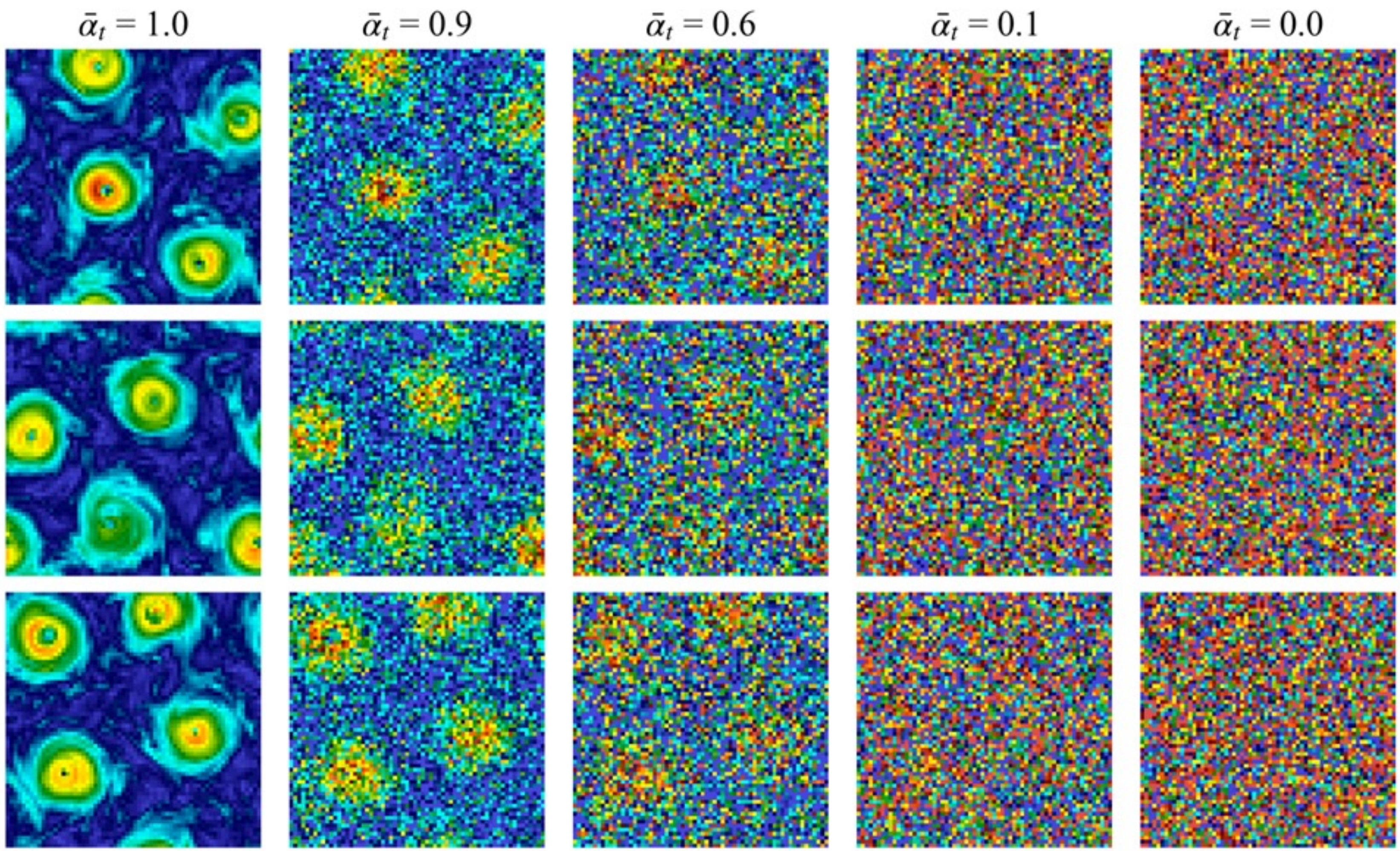


2 invariants (positive definite)

$$\Omega = \int d^3x\, \omega \cdot \omega \quad E = \int d^3x\, \mathbf{v} \cdot \mathbf{v}$$



$\bar{\alpha}_t = 1.0$
$\bar{\alpha}_t = 0.9$
$\bar{\alpha}_t = 0.6$
$\bar{\alpha}_t = 0.1$
$\bar{\alpha}_t = 0.0$