Extreme Events in the Dispersions of Two Neighboring Particles Under the Influence of Fluid Turbulence

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We present a numerical study of two-particle dispersion from point sources in three-dimensional incompressible homogeneous and isotropic turbulence at Reynolds number Re $\simeq$ 300. Tracer particles are emitted in bunches from localized sources smaller than the Kolmogorov scale. We report the first quantitative evidence, supported by an unprecedented statistics, of the deviations of relative dispersion from Richardson's picture. Deviations are due to extreme events of pairs separating much faster than average, and of pairs remaining close for long time. The two classes of events are the fingerprints of complete different physics, the former dominated by inertial subrange and large-scale fluctuations, and the latter by dissipation subrange. A comparison of the relative separation in surrogate white-in-time velocity field, with correct viscous-, inertial-, and integral-scale properties, allows us to assess the importance of temporal correlations along tracer trajectories.

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The relative separation of pairs of fluid particles by turbulence was first addressed by Richardson [1–3]. The main question is simple and fundamental: given a pair of particles released at time t_0 and at a small separation $\boldsymbol{r}_0$ (smaller of the Kolmogorov dissipative scale, η), what is the probability to find them at a distance r at a later time t? In the case of isotropic and homogeneous turbulence (HIT), the probability density function (PDF) $P(\boldsymbol{r}, t|\boldsymbol{r}_0, t_0)$ of pair separation depends on the amplitude of $\boldsymbol{r}$ only. Moreover, asymptotically it should become independent of the initial condition. The knowledge of $P(r, t)$ is of utmost importance for many studies, and it constitutes a highly nontrivial statistical problem. It is intrinsically nonstationary as it connects velocities at particle positions, along their whole past history [4]. Richardson proposed to model particle separation in the inertial range $\eta \ll r \ll L_0$ as a diffusive process with an effective turbulent diffusivity, estimated empirically to follow a 4/3 law: $D_{\mathrm{Ric}}(r) = \frac{d\langle r^2\rangle}{dt} \sim \beta r^{4/3}$. Here L_0 is the large scale of the flow and $\beta = k_0\epsilon^{1/3}$, where k_0 is a dimensionless constant and ϵ the turbulent kinetic energy dissipation. It is easy to connect Richardson's work with Kolmogorov's 1941 theory by means of the dimensional estimate [4,5],

$$D_{\mathrm{Ric}}(r) \sim \tau(r)\langle(\delta_r v)^2\rangle, \tag{1}$$

where $\tau(r) \sim \epsilon^{-1/3} r^{2/3}$ is the eddy turnover time at scale r and $\langle(\delta_r v)^2\rangle = C_0\epsilon^{2/3} r^{2/3}$ is the second-order Eulerian longitudinal structure function. The resulting long-time growth of the mean squared separation is

$$\langle r^2(t)\rangle = g\epsilon t^3, \tag{2}$$

where g is the Richardson constant uniquely determined in terms of k_0 [6–9]. Many studies [8,10–13] have focused on the subject, including extensions to the case of particles with inertia [14,15]. Richardson's picture definitely captures some important features of turbulent dispersion, e.g., concerning events with a typical separation of the order of the mean. However, fundamental questions exist on the possibility to correctly predict extremal events, i.e., pairs with separation much larger or smaller than $\langle r^2(t)\rangle^{1/2}$. Richardson's approach can be rephrased as the evolution of tracers in a stochastic Gaussian, homogeneous, incompressible, and isotropic velocity field, δ-correlated in time, with a two-point longitudinal correlation $D_{\parallel}(r)$ [16]. Under this assumption, the evolution of $P(r, t)$ is closed and local [4,16],

$$\partial_t P(r, t) = r^{-2}\partial_r r^2 D_{\parallel}(r)\partial_r P(r, t). \tag{3}$$

Whenever $D_{\parallel}(r) = D_0 r^{\xi}$, this equation, with $0 \le \xi < 2$ and $P(r, t_0) \propto \delta(r - r_0)$, can be solved analytically [17], and provides the celebrated asymptotic large-time solution (independent of r_0),

$$P_{\mathrm{Ric}}(r, t) \propto \frac{r^2}{\langle r^2(t)\rangle^{3/2}} \exp\left\{-b\left(\frac{r}{\langle r^2(t)\rangle^{1/2}}\right)^{2-\xi}\right\}. \tag{4}$$

Here b is a constant, uniquely determined by D_0 [17]. In such idealized scaling scenarios, tracer pairs separate in an explosive way, forgetting their initial separation $\langle r^2(t)\rangle \propto t^{2/(2-\xi)}$, which reproduces Richardson's expression for $\xi = 4/3$.

When the turbulent flow is differentiable, i.e., $\xi = 2$, the PDF takes the log-normal form $P(r, t|r_0, 0) \propto \exp\{-[\log(r/r_0) - \lambda t]^2/(2\Delta t)\}$, where λ is the first Lyapunov exponent and Δ is connected to fluctuations of the strain matrix [4]. In the latter case, particles separate exponentially and the memory of the initial separation r_0 remains at all times. The rate of separation strongly fluctuates from point to point and from time to time, being connected to the fluctuations of the Lyapunov exponents [4,18].

Particle behavior in real flows can deviate from Richardson's picture due to several reasons: (i) temporal correlations of the underlying velocity fluid [5,16,19], (ii) non-Gaussian velocity fluctuations, (iii) ultraviolet (UV) effects induced by the dissipation subrange, and (iv) infrared (IR) effects induced by a large-scale cutoff. These last two are connected with finite Reynolds effects [20].

The goal of this Letter is twofold. First, we want to understand and quantify the rate of occurrence of rare extreme events, i.e., the events of pairs that separate much more or much less than $\langle r^2(t)\rangle^{1/2}$. Second, we aim to assess the importance of temporal correlations for both pair statistics in general, and extreme events in particular. We performed a series of direct numerical simulations (DNS) of HIT seeding the fluid with bunches of tracers, emitted in different locations, to reduce spatial correlations. Each bunch is emitted within a small region of size $\sim\eta$, in puffs of 2×10^3 tracer particles each. In a single run, there are 256 of such sources, emitting about 200 puffs, with a frequency comparable with the inverse of the Kolmogorov time. We performed ten different runs, following a total of 4×10^{11} pairs, reaching an unprecedented statistics. In Fig. 1, we illustrate the complexity of the problem. We first notice the abrupt transition in particle dispersion occurring at about $\sim 10\tau_\eta$ after the emission, when most of the pairs reach a relative distance of the order of $\sim 10\eta$, and the beginning of an explosive separation in the manner of Richardson is observed. At a later stage, there are many pairs with relative separation of the order of the box size $\sim 1000\eta$, even though the mean separation is much smaller at those time lags. On the contrary, in the inset of Fig. 1 we show an example of a bunch with an anomalous history due to tracers that travel close—at mutual distance of the order of η—for very long times. This happens when pairs are injected in a space location where the underlying fluid has a small local stretching rate.

To quantify this phenomenology, we show in Fig. 2 the right and left tails of $P(r, t)$ at different time lags. The top panel shows that the fastest events have an exponential-like tail. A cutoff separation, $r_c(t)$ is identified when a sharp change in the slope is observed. The scale $r_c(t)$ is connected to pairs that are able to separate "very fast." It indicates the existence of pairs experiencing a persistently high relative velocity limited in amplitude by the root mean squared single point value, $v_{\rm rms}$ [21]. To support this statement, we show in the inset the evolution of $r_c(t)$ which is in good agreement with the linear behavior obtained using

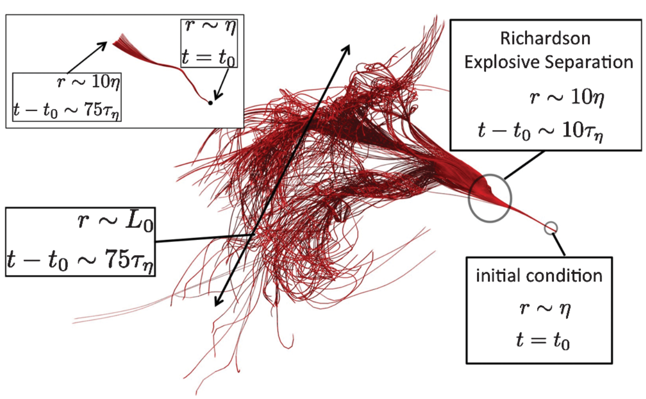

FIG. 1 (color). Typical time history up to $t = 75\tau_\eta$ of a bunch emitted from a source of size $\sim\eta$. Inset: Time history for the same duration of a bunch emitted in a different location, and which does not separate. DNS are performed on a cubic fully periodic grid at 1024^3 collocation points with a pseudospectral code, at a Reynolds-Taylor number $\mathrm{Re}_\lambda \sim 300$. For further details on the numerics; see Ref. [8].

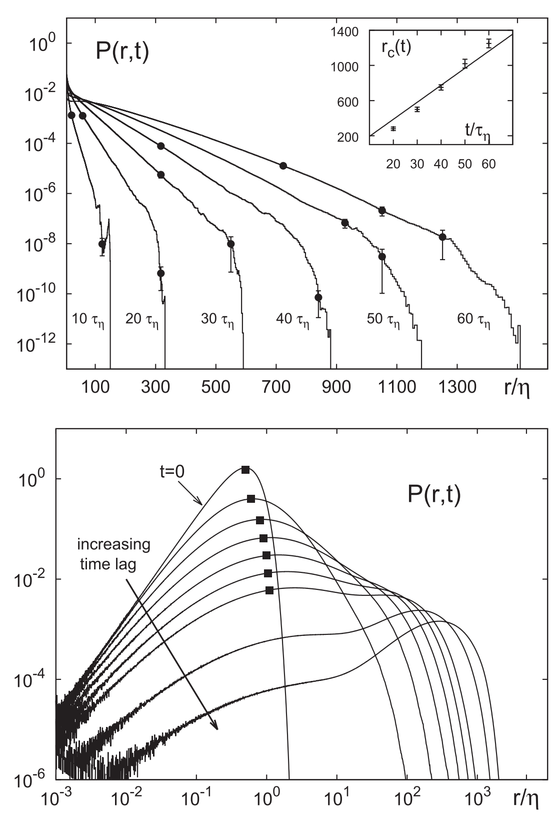

FIG. 2. Top: log-lin plot of $P(r, t)$ at different times after the emission. For selected values of r, we show error bars, estimated from the statistical spread of different runs. Inset: evolution of the cutoff scale $r_c(t)$; the continuous line represents the ballistic motion $\propto v_{\rm rms}t$. Bottom: log-log plot of $P(r, t)$ for $t = (10, 20, 30, 40, 50, 60, 70, 90, 120)\tau_\eta$. The black squares indicate the peaks observed for small separations.

$v_{\rm rms}$ as the traveling speed. Events beyond $r_c(t)$ are rare, and can be detected with high statistics only.

The opposite limit of "very slow" events is also remarkable (see Fig. 2). Here, we observe a bimodal shape for $P(r, t)$ at almost all times: the left tail of the pairs with mutual distance $r < \eta$ remains populated for a period of time up to $\sim$60–70τ_η, which is of the order of the large-scale eddy turnover time T_L. The pairs emitted in regions with a small stretching rate tend to stay together. We could empirically find that this tail can be well fitted by a log-normal distribution (not shown), as for the case of a spatially smooth flow, shortly correlated in time. Such strong persistence at subdiffusive scales cannot be brought back to small-scale clustering effects as those observed in the dynamics of inertial particles [22]. It must be strongly sensitive to the intermittent nature of the turbulent stretching rate with higher than Gaussian probability to have small and large events.

In Fig. 3, we plot the same data of Fig. 2 but rescaled in terms of the variable $r_n(t) = r/\langle r^2(t)\rangle^{1/2}$, and compared against the asymptotic prediction Eq. (4). Here, the deviations from $P_{\rm Ric}(r, t)$ at large scales for all times are evident. A more stringent test is obtained by showing these same PDFs, but restricted to the scales in the inertial subrange $30\eta < r < 300\eta$ (inset). Clearly, Eq. (4) is not satisfied. Previous studies could access events only up to $r/\langle r^2(t)\rangle^{1/2} < 3$ (see Ref. [3]). Our study improves by 5 orders of magnitudes (in probability) the intensities of detectable events, thus allowing us to highlight strong deviations from Richardson's shape. Large discrepancies can be measured also on the left tails of $P(r, t)$, associated to very slow separating pairs (see also below).

Such departures from the ideal self-similar Richardson distribution needs to be better quantified, either in terms of finite Reynolds effects (breakup of self-similarity of the turbulent eddy diffusivity) or in terms of the neglected temporal correlations, or both.

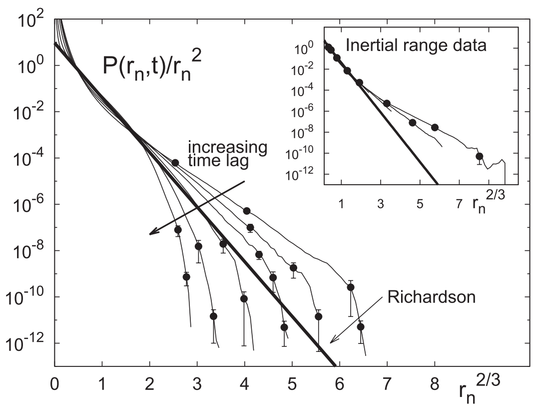

FIG. 3. Log-lin plot of $P(r_n, t)$ versus the rescaled variable r_n (see text) for $t = (20, 30, 40, 60, 90, 120)\tau_\eta$. The distribution $P(r_n, t)$ has been divided by a factor r_n^2 to highlight the large separation range. The Richardson prediction, Eq. (4), becomes time independent if rescaled in this way (solid curve). Inset: PDFs plotted only for separations r_n that, at time lag $t \in [10{:}120]\tau_\eta$, belong to the inertial subrange.

To assess the importance of the former, we have integrated Eq. (3) using an effective eddy diffusivity $D^{\rm eff}(r)$, which improves Eq. (1) by including viscous and large-scale behaviors,

$$\begin{aligned} D_\parallel^{\rm eff}(r) &\sim r^2 & r &\ll \eta \\ D_\parallel^{\rm eff}(r) &\sim r^{4/3} & \eta &\ll r \ll L_0 . \\ D_\parallel^{\rm eff}(r) &\sim \text{const} & r &\gg L_0 \end{aligned} \tag{5}$$

A widely used fitting formula that reproduces well the Eulerian data, and that matches the expected UV and IR scaling for both $\tau(r)$ and $\langle(\delta_r v)^2\rangle$, is obtained by the following equation [23]:

$$\langle(\delta_r v)^2\rangle = c_0 \frac{r^2}{[(r/\eta)^2 + c_1]^{2/3}}\left[1 + c_2\left(\frac{r}{L_0}\right)^2\right]^{-1/3} \tag{6}$$

supplemented with a similar expression for the eddy turnover time, $\tau(r) = \frac{\tau_\eta}{[(r/\eta)^2+c_1]^{-1/3}}[1 + d_2(r/L_0)^2]^{-1/3}$. The dimensionless parameters c_0, c_1, c_2 are extracted from the Eulerian statistics, while the parameter d_2 is fixed such as to correctly reproduce the evolution of the mean square separation $\langle r^2(t)\rangle$ over a time range $\tau_\eta \le t \le T_L$ (see Fig. 4). Despite the excellent agreement for $\langle r^2(t)\rangle$ shown in Fig. 4, the solution to the diffusive equation (3) using $D^{\rm eff}(r)$ does not match the data in the far tails as shown in Fig. 5. Self-similarity is broken by the introduction of UV and IR cutoffs in Eq. (5), and therefore $P_{\rm eff}(r, t)$ no longer rescales at different times as observed in real turbulent flows. For large times, the agreement with the DNS data is qualitatively better, but still quantitatively off, particularly when focusing on the sharp change at $r_c(t)$ which is still absent in the evolution given by Eq. (5). This is a key point, showing that to reproduce the observed drop

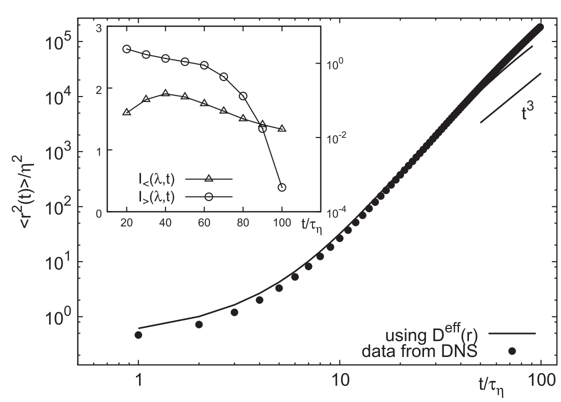

FIG. 4. Log-log plot of $\langle r^2(t)\rangle$ from DNS data, and from the diffusive evolution with eddy diffusivity (5). Inset: time evolution of the relative probability to observe a large excursion, $I_>(\lambda, t)$ (right y scale), or small excursion $I_<(\lambda, t)$ (left y scale) for $\lambda = 3$.

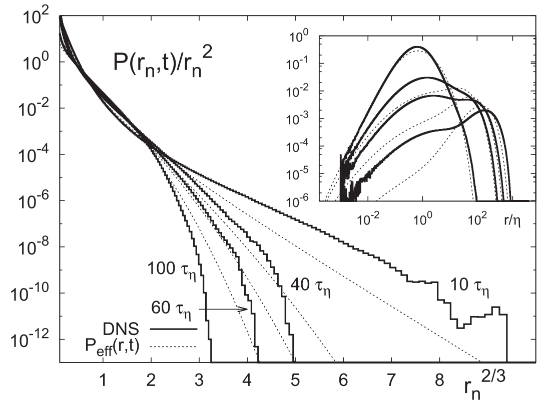

FIG. 5. Lin-log plot of $P_{\rm eff}(r, t)$ as obtained from the integration of Eq. (4) with $D^{\rm eff}(r)$ (dashed) and the DNS data (solid line). Inset: log-log plot to highlight the slowest events (left tail).

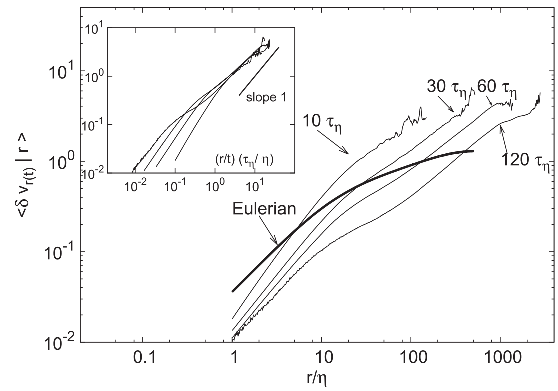

FIG. 6. Comparison between conditional Lagrangian (thin lines) and Eulerian (thick line) velocity increment moment. Inset: Lagrangian quantities plotted versus $(r/\eta)/(t/\tau_\eta)$.

at $r_c(t)$, it is not enough to impose a saturation of $D_{\parallel}^{\rm eff}$, for large r. The behavior of pair dispersion must then be either dependent on the nature of temporal correlations or on the presence of a finite propagation speed induced by the $v_{\rm rms}$ in the flow (see the inset of Fig. 2).

Concerning the small separation tail, $P_{\rm eff}(r, t)$ presents a slowly evolving peak for $r \leq \eta$, but quantitative agreement is not satisfactory (see the inset of Fig. 5). We explain it as the effect of assuming a Gaussian statistics, which is blatantly wrong because of turbulent small-scale intermittency.

To further quantify the departure of the modified Richardson description from the real data, we measured the cumulative probability to have a couple at large separation $r^* = \lambda\langle r^2(t)\rangle^{1/2}$ normalized with the same quantity evaluated from Eq. (4) using Eq. (5), namely, $I_>(\lambda, t) = \int_{r^*}^{\infty} drP(r, t)/\int_{r^*}^{\infty} drP_{\rm eff}(r, t)$. Similarly, to evaluate the differences for small separation events, we use $r^* = 1/\lambda\langle r^2(t)\rangle^{1/2}$ and define $I_<(\lambda, t) = \int_0^{r^*} drP(r, t)/\int_0^{r^*} drP_{\rm eff}(r, t)$. The results are shown in the inset of Fig. 4 for $\lambda = 3$. Concerning $I_<(\lambda, t)$, the evolution using $P_{\rm eff}(r, t)$ underestimates by a factor 2, at small time lags, the importance of the small scale trapping, i.e., does not capture the strong intermittency of the regions where we have a small stretching rate. Only for very large times $\sim 100\tau_\eta$, the left tail becomes comparable with the real ones. Concerning $I_>(\lambda, t)$, we measure first an underestimate of large separation events and later a strong overestimate; i.e., δ-correlation does not capture the presence of $r_c(t)$.

To quantify the importance of temporal correlations, we compare in Fig. 6 the velocity increments conditioned on particle distance r, $S_{\rm Lag}(r, t) = \langle(\delta_{r(t)}v_i \cdot \hat{r}_i)|r(t) = r\rangle$, with its Eulerian equivalent, $S_{\rm Eul}(r) = \langle|\delta_r v_i \cdot \hat{r}_i|\rangle$. Fast separating pairs have a typical velocity difference higher than the Eulerian one. Such velocity fluctuations can be estimated as $\sim r/t$, and Lagrangian structure function should superpose when plotted versus r/t for different t and for large r (see Fig. 6).

We presented the first high-statistics numerical study able to explore rare events in turbulent dispersion. Both "fast" and "slow" separations depart from the Richardson inertial and idealized behavior. A step forward is obtained by maintaining the assumption of δ-correlation and Gaussian statistics, but with an improved effective eddy-diffusivity kernel that correctly takes into account viscous- and integral-scale physics. To progress further, one needs to relax the Gaussianity assumption for small-scale and the δ-correlation for integral-scale separations. The former is crucial to correctly estimate small-stretching rate events that lead to slow separations. An attempt following the latter direction has been proposed in [24] using a Langevin process for the relative particles velocity, with different correlation times at different scales. Alternatively, a memory kernel can be used to avoid unphysical events with infinite speed [19], otherwise present in a δ-correlated velocity field (e.g., one-dimensional telegraph-equation models show similar maximum speed events [21]). A change in the far separation range for $P(r, t)$ was predicted in Ref. [16], where the turbulent diffusivity for extreme events $t \ll r/\delta_r v$ was replaced with its "ballistic" estimate $\langle(\delta_r v)^2\rangle^{1/2}t$. In such cases, the far tail should follow a stretched exponential with a $r^{4/3}$ shape.

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