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On the Intermittent Energy Transfer at Viscous Scales in Turbulent Flows.

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Abstract. – In this letter we present numerical and experimental results on the scaling properties of velocity turbulent fields in the range of scales where viscous effects are acting. A generalized version of extended self-similarity capable of describing scaling laws of the velocity structure functions down to the smallest resolvable scales is introduced. Our findings suggest the absence of any sharp viscous cut-off in the intermittent transfer of energy.

The word anomalous scaling (AS) usually refers to scaling laws in a physical system which cannot be deduced from naive dimensional arguments. It is always a challenging problem in physics to understand the origin of anomalous scaling and to formulate a predictive theoretical framework to compute the anomalous-scaling exponents.

Among the many physical systems showing anomalous scaling, fully developed three-dimensional turbulence (FDT) has been widely investigated in the last few years (see [1] for a recent overview of the experimental and theoretical state of the art). According to Kolmogorov 1941 theory [2], the small-scale statistical properties of FDT obey the relation

$$\langle |\delta v(r)|^p \rangle \sim A_p \left(\frac{r}{L}\right)^{p/3} v_0^p \sim A_p \varepsilon^{p/3} r^{p/3} , \tag{1}$$

where $\delta v(r) = v(x+r) - v(x)$ is the difference of velocity at scale r, v_0 is the r.m.s. velocity at the integral scale L, ε is the mean energy dissipation and the A_p's are dimensionless constants. Equation (1) is not satisfied both in real experiments and numerical simulation.

Indeed, one has to replace it by anomalous (also known as intermittent) scaling

$$\langle |\delta v(r)|^p \rangle \sim B_p \left(\frac{r}{L} \right)^{\zeta(p)} v_0^p , \tag{2}$$

where $\zeta(p)$ is now a non-linear function of its argument.

At variance with expression (1), the scaling (2) is anomalous in the sense that it cannot be deduced by naive dimensional counting. In order to get a more precise measurement of the $\zeta(p)$ exponents and to highlight the anomalous scaling, it has been proposed in [3, 4] to look at the self-scaling properties of the velocity structure functions, namely

$$\langle |\delta v(r)|^p \rangle \sim \langle |\delta v(r)|^q \rangle^{\beta(p,q)} . \tag{3}$$

This new way of looking at the scaling properties has been tested in many different experimental and numerical instances [5]. In all cases, when small-scale homogeneity and isotropy were satisfied, a dramatic improvement in the width of the scaling region was observed. This almost universal property of turbulent flows was then called Extended Self-Similarity (ESS). ESS must be interpreted as the signature of some non-trivial universal physics happening at the transition between the inertial and viscous scale. It tells us that, by using the appropriate functional form, scaling is present also at scales where in principle viscous effects should already be important.

The aim of this letter is to present a generalized version of ESS (G-ESS) which turns out to be much more universal and allows us to draw a concrete theoretical framework of the energy cascade down to the smallest resolvable scale, *i.e.* in a region where no anomalous scaling was supposed to be detected.

The physical outcome of our findings is that whatever is the mechanism responsible for anomalous scaling in FDT, this mechanism is acting also at extremely small scales and, within experimental errors, no evidence of a cut-off (due to dissipation) is observed.

Let us first introduce the dimensionless structure functions

$$G_p(r) = \frac{\langle |\delta v(r)|^p \rangle}{\langle |\delta v(r)|^3 \rangle^{p/3}} . \tag{4}$$

According to Kolmogorov theory, $G_p(r)$ should be a constant both in the inertial range and in the dissipative range, although the two constants are not necessarily thought to be the same. Because of the presence of anomalous scaling, $G_p(r)$ are no longer constants in the inertial range.

Our main point consists in studying the self-scaling properties of the dimensionless structure functions, namely for any p and q, we consider the scaling relation

$$G_p(r) = G_q(r)^{\varrho(p,q)} , \tag{5}$$

where we have by definition

$$\varrho(p,q) = \frac{\zeta(p) - p/3\zeta(3)}{\zeta(q) - q/3\zeta(3)} , \tag{6}$$

$\varrho(p,q)$ is given by the ratio between deviations from the K41 scaling.

Equation (6) is certainly satisfied for all cases where an anomalous scaling is observed for the velocity structure functions. In particular, eq. (6) is also satisfied when ESS is observed. On the other hand, it is reasonable to imagine that the velocity field becomes laminar in the subviscous range, $\langle |\delta v(r)|^p \rangle \sim r^p$, still preserving some intermittent degree parametrized by

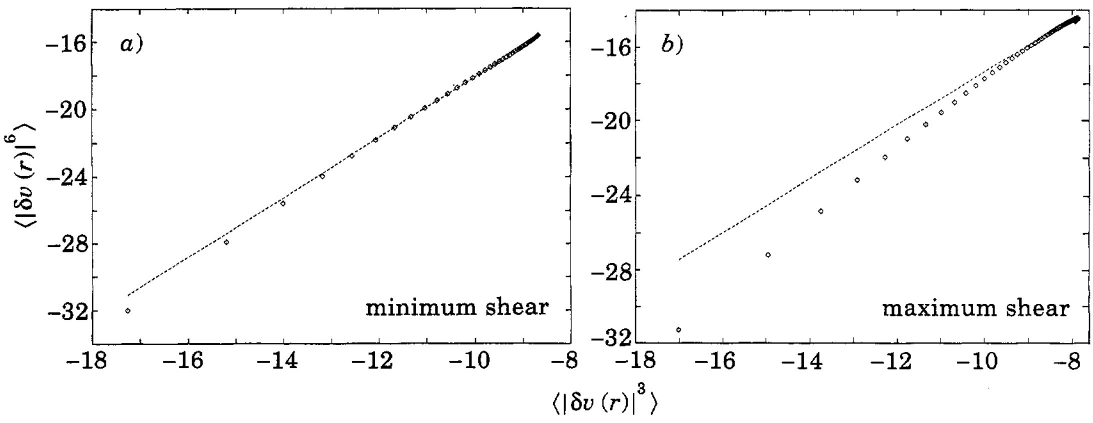

Fig. 1. – *a*) Log-log plot showing ESS scaling for the longitudinal structure functions, $|\delta v(r)|^6$ *vs.* $|\delta v(r)|^3$. Data are taken from a direct numerical simulation of a shear flow at $Re_\lambda = 40$. Each point corresponds to a space separation of a Kolmogorov scale. The computation of structure functions is performed at points where the shear is minimum. The dashed line is the best fit for the slope in the scaling region. *b*) The same as *a*) but for points where the shear is maximum. At variance with the previous case ESS is not observed.

the ratio between corrections to K41 theory. If this is the case, one can argue that $\varrho(p, q)$ is the only quantity that can stay constant along all the cascade process: from the integral to the subviscous scales.

We want to support our previous discussion by analysing a data set obtained from a direct numerical simulation of 3-dimensional Navier-Stokes equations for a Kolmogorov flow (see [6] for technical details).

The flow is forced such that the stationary solution has a non-zero spatial-dependent mean velocity $\langle \boldsymbol{v}(\boldsymbol{x})\rangle = \hat{x}\sin((8\pi/L)z)$, where $\hat{x}$ is the versor in the direction x, and L is the integral scale.

In fig. 1*a*) and *b*) we show the standard ESS analysis by plotting $\langle|\delta v(r)|^6\rangle$ *vs.* $\langle|\delta v(r)|^3\rangle$ for two specific levels z_a and z_b, where z_a was chosen at minimum shear and z_b at maximum shear (in this case $\langle\ldots\rangle$ must be interpreted as averages over time integration at fixed z-level). The Re_λ number of the simulation was 40 and no scaling laws were present if examined as a function of the physical scale r. Nevertheless, it is clear from fig. 1*a*) that ESS is observed for the case of minimum shear and it is not observed for the case of maximum shear (fig. 1*b*)). Violations of ESS have already been reported in other cases where strong shear effects were argued to be relevant [7, 4].

On the other hand, the self-scaling (6) (hereafter referred to as G-ESS) works perfectly well at all resolvable scales, as can be seen in fig. 2.

The analysis done for the Kolmogorov flow has been repeated for many different experimental set-ups [3, 8, 9], done at different Reynolds number, and for some direct numerical simulation, with and without large-scale shear. In fig. 3 we have plotted the scaling of $G_6(r)$ *vs.* $G_5(r)$ for all cases previously cited. As one can see the straight-line behaviour is very well supported. Within experimental errors (of the order of 3%) no deviations from the scaling regime are detected. Similar results are obtained, using different $G_p(r)$ and $G_q(r)$.

Finally, let us remark a possible theoretical interpretation of G-ESS based on a recent model proposed in [10]. According to [10], the anomalous exponents $\zeta(p)$ are well fitted by the formula

$$\zeta(p) = h_0 p + d_0 (1 - \beta^{p/3}), \tag{7}$$

where h_0 and d_0 are free parameters describing the geometric nature of the coherent structures in turbulent flows, and β is defined such that $\zeta(3) = 1$. Using (7) and (6), one gets

$$\varrho(p, q) = \frac{(1 - \beta^{p/3}) - (p/3)(1 - \beta)}{(1 - \beta^{q/3}) - (q/3)(1 - \beta)}. \tag{8}$$

The interpretation of G-ESS within the model proposed by [10] can be easily obtained by (8). Indeed, one can think that the geometric characteristic of the coherent structures (defined in terms of h_0 and d_0) can be scale dependent while the non-linear intermittent energy transfer (parametrized in [10] by β) is scale independent. The possible outcomes or failures of this interpretation are left for future works.

In a more detailed version of this study [11] we will discuss how it is possible to reconcile the idea of multiplicative cascades with this continuous energy transfer from the inertial range to the viscous range. We will present also numerical evidences that this new scaling behaviour is in disagreement with previously proposed ideas of a statistical-dependent viscous cut-off as predicted in all the standard multiplicative multifractal models [12].

Our results may have theoretical and applied implication. For instance, the presence of

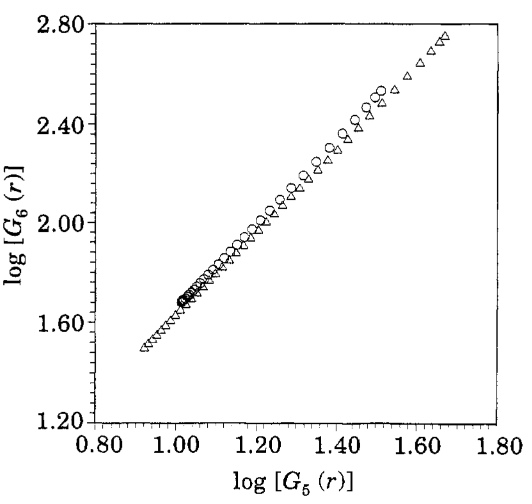

Fig. 2.

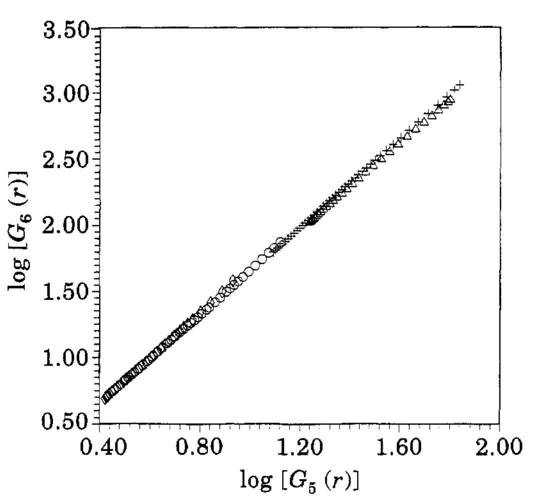

Fig. 3.

Fig. 2. – Log-log plot of $G_6(r)$ *vs.* $G_5(r)$ for the shear flow for both cases of maximum shear (circles) and minimum shear (triangels). At variance with the standard ESS analysis, we can now observe a clean scaling behaviour which extends down to the smallest resolvable scale.

Fig. 3. – Log-log plot of $G_6(r)$ *vs.* $G_5(r)$ for different laboratory and numerical experiments. Data taken in a wake behind a cylinder, where standard ESS was not observed [3] (crosses). Data taken from the region with log-profile of a boundary layer (courtesy of G. Ruiz Chavarria) where standard ESS was not observed (circles). Data taken from a direct numerical simulation of thermal convection [8] where standard ESS was observed (squares). Data from a direct numerical simulation of a channel flow where standard ESS was not observed [9] (triangles).

intermittent fluctuations at all scales might cast serious doubts on the validity of renormalized perturbative expansion of the NS equations which are usually based on perturbative expansion around the linearized equations.

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