RENORMALIZATION GROUP STUDY OF THE THREE STATE THREE DIMENSIONAL POTTS MODEL

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We make a renormalisation group study with the finite size real space renormalization group (FSRSRG) technique of the order of the transition in the three dimensional three state Potts model with nearest neighbor and next to nearest neighbour couplings. Our results provide evidence that the order–disorder transition from a high temperature phase to a low temperature phase is discontinuous not only in the region characterized by both couplings being positive but also when the next to nearest coupling is appreciably antiferromagnetic. In both cases the correlation length exponent, previously not known, converges to the value expected for a first order first transition but only for large volumes, especially in the antiferromagnetic case.

Recent results [1] in QCD have determined a renewed interest in the properties of the three state ($q=3$) Potts model [2]. Approximate arguments have been proposed [3], according to which the SU(3) model in (3+1) dimensions and a three dimensional spin model with Z(3) discrete symmetry should display the same critical behaviour.

In spite of the fact that the corresponding spin model itself is much simpler than the SU(3) model used in QCD, exact results concerning its phase structure are nowadays still scarse. Even less clear is the situation when two kinds of couplings are present. In this letter, we consider the three state Potts model with nearest neighbour J_1 and next to nearest neighbour J_2 couplings in three dimensions, governed by the following hamiltonian:

$$H=J_1 \sum_{\substack{\langle ij\rangle \\ \text{nn}}} \sigma_i\sigma_j + J_2 \sum_{\substack{\langle ij\rangle \\ \text{nnn}}} \sigma_i\sigma_j\,, \qquad (1)$$

where the discrete Potts spin variable σ_i can only take on three different values. The first sum in eq. (1) runs over the nearest neighbour (nn) spins, at distance 1 and the second over the next to nearest neighbour (nnn), at distance $\sqrt{2}$ in lattice units. Their number is respectively 6 and 12 for the cubic lattice considered here.

A mean field analysis predicts that the transition is always first order, due to the presence of a nonvanishing cubic term in the free energy. A recent study [4] based on a high precision Monte Carlo (MC) simulation, seems to confirm that this picture is correct in the case of $J_2=0$. However, some authors claimed [5] that the transition is second order for some negative values of J_2. No attempt was made to measure the critical exponents. In the present paper we apply the finite size real space renormalisation group method (FSRSRG) [6] to the study of the order–disorder transition in the model described by eq. (1).

The FSRSRG combines the advantages of the MC renormalization group with those of the finite size

scaling [7]. It blocks the original spin system to a system containing a fixed number of block spins. The critical exponents are obtained from the analysis of the flow of the block couplings as a function of the total lattice size.

Indeed, in the proximity of a phase transition and for values of L sufficiently large, the renormalized couplings relative to systems of different sizes approach a common value K_i^*, which is independent of L. This defines a matching point, about which one proceeds to linearize the RG transformation and to extract the value of the critical exponents. For a detailed account of the method we refer to the original papers [6] where it was tested for the 2D Ising, Potts, XY and Heisenberg models.

The estimate of the critical point exponents requires the knowledge of the renormalized couplings and of their derivatives with respect to the bare couplings J_i. In order to compute the thermal critical exponent $y_T = 1/\nu$ we defined the following three block variables or renormalized couplings:

$$K_1(L) = \left\langle \sum_{\substack{\langle ij \rangle \\ \mathrm{nn}}} \Sigma_i \Sigma_j \right\rangle , \tag{2a}$$

$$K_2(L) = \left\langle \sum_{\substack{\langle ij \rangle \\ \mathrm{nnn}}} \Sigma_i \Sigma_j \right\rangle , \tag{2b}$$

The blocked spins Σ_i are obtained by a majority rule on a cube containing $(L/2)^d$ site variables and the indices i, j run over the 2^d sites of the blocked lattice. The blocked coupling K_1 represents the nearest neighbour average interaction energy in the small lattice, whereas K_2 is the next to nearest neighbour average interaction energy.

A third coupling was also defined by means of a different kind of blocking transformation of the original L^d sites lattice into a lattice containing only two cells [8]. To define the transformation one starts by dividing the larger lattice into 2^d equivalent sublattices of volume $(L/2)^d$ and next grouping together the $L^d/2$ sites obtained by considering the system formed by one sublattice arbitrarily chosen and the three sublattices with one face in common with the latter. This procedure defines two non-intersecting subsystems. By applying the majority rule to each subsystem, we finally obtain the new block spin variables Σ_A and Σ_B. The coupling $K_3(L)$ is defined as

$$K_3(L) = \langle \Sigma_A \Sigma_B \rangle . \tag{2c}$$

To improve the statistical performance we computed K_3 in the four possible independent ways.

It is straightforward to show that the derivatives of these renormalized couplings with respect to J_1 and J_2 can be expressed in terms of connected averages of products of block spins Σ and original spins σ.

The thermal critical index is calculated by using the formula

$$y_T^{i,\beta} = \ln\left(\frac{\mathrm{d}K_i(L_1)/\mathrm{d}J_\beta}{\mathrm{d}K_i(L_2)/\mathrm{d}J_\beta} \right) [\ln(L_1/L_2)]^{-1} . \tag{3}$$

The scaling picture of phase transitions predicts that, at a first order phase transition associated with a couplings flow, the thermal exponent is equal to the dimensionality of the system, d [9].

We monitored the order of the transition by measuring the value of the exponent y_T as J_1 and J_2 varied. We carried out two different sets of runs. In the first set we varied the bare couplings J_β's, while keeping $J_1 = J_2$. In the second set instead we kept $J_1 = 1$ fixed and varied J_2. These two choices were sufficient to guarantee an effective crossing with the transition line with a sharp and statistically stable couplings flow around the fixed point.

In the case $J_1 = J_2$ we can write H as

$$H = J\left(\sum_{\substack{\langle ij \rangle \\ \mathrm{nn}}} \sigma_i \sigma_j + \sum_{\substack{\langle ij \rangle \\ \mathrm{nnn}}} \sigma_i \sigma_j \right) , \tag{4}$$

and then we consider the J derivatives in eq. (3). In the $J_1 = 1$ fixed case we consider the J_β derivative for each coupling, and we dispose of six quantities in order to compute y_T in eq. (3).

All the averages were calculated by means of MC simulation of cubic systems of linear sizes $L = 8$, 12, 16, 20, 24; in the case $J_1 = 1$ we also made a run with $L = 28$ and $L = 32$. We use multi-spin coding to store eight independent lattices in a single 32 bit word. In all cases, in order to reduce the thermalization time, we started from an equilibrium configuration obtained in a previous run at a slightly different value of the parameters. The number of thermalisation and measurement sweeps we made at each volume are reported in table 1. The set of measurements naturally

Table 1

L	Thermalisation sweeps	Measurement sweeps
8	50 000	1 000 000
12	75 000	1 000 000
16	125 000	1 000 000
20	300 000	1 920 000
24	300 000	1 920 000
28	390 000	1 280 000
32	450 000	3 120 000

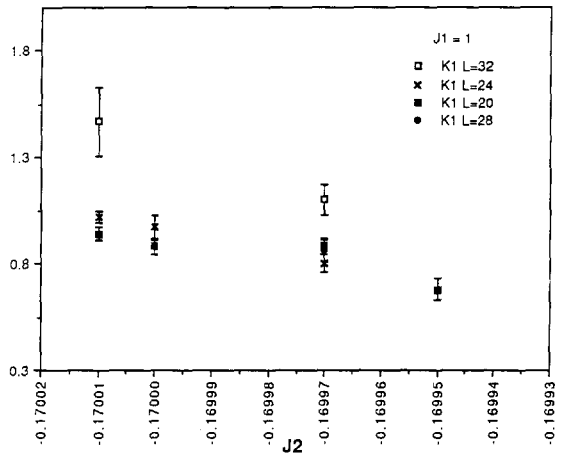

Fig. 1. Flux of the renormalized coupling K_1 against J_2 in the AF case.

splits into the eight independent multi-coded lattices and the errors are estimated from the fluctuations of these eight samples. In the cases with large statistics like for large lattices, we have further subdivided each lattice sample into clusters to check against thermalisation effects and the results among different clusters turned out to be stable and consistent. We always started from a random configuration: by monitoring the average energy of the system we have observed several flips between two values confirming the first order nature of the transition.

Figs. 1 and 2 display the flow of the couplings K_1 as a function of J and L for the case $J_1=1$ relative to the lattices of size $L=8$, 12, 16 and $L=20$, 24, 28, 32, respectively. For large volumes, the sensitivity of the flow to the tuning of the original couplings increases and one needs an expanded horizontal scale. The fixed point values shift with increasing volumes as expected. In the first case we estimate

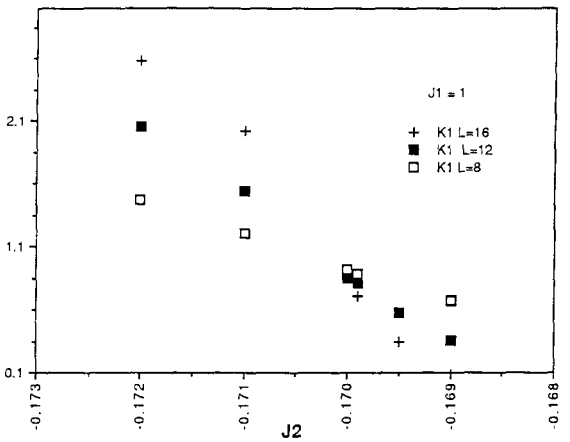

Fig. 2. Same quantity as in fig. 1 but for larger volumes.

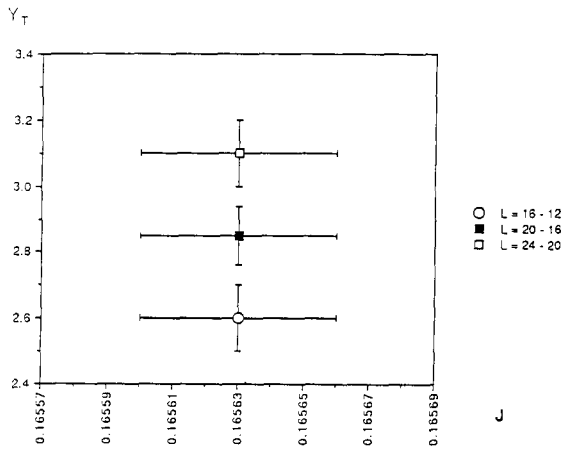

Fig. 3. Thermal exponent y_T obtained from eq. (3) for $J_1=J_2$ by considering three different pairs of lattice sizes.

$$J^*=0.16563\ (3)\,, \tag{5a}$$

and in the second

$$J_2^*=-0.16998\ (2)\,. \tag{5b}$$

For $J_1=J_2$ our results provide further support to the conjecture that the transition is first order in the region of positive couplings. Fluctuations are not sufficient to modify the mean field scenario, which also predicts a discontinuous transition. For lattice sizes greater than $L=12$ we consistently measure a thermal exponent $y_T=3.0\pm0.1$ as can be seen from fig. 3 where we plot the thermal exponent in the J matching region and for different lattice sizes.

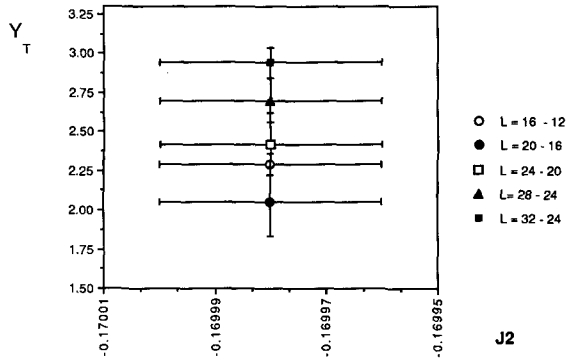

Fig. 4. Thermal exponent y_T for the AF case for various pairs of lattice sizes.

In the $J_1 = 1$ and $J_2 < 0$ case we find that the first order character of the transition is harder to determine: the exponent drifts up to large lattices where it stabilises at a value which is 3.2 ± 0.25, as shown in fig. 4. The results for the transition point, and also for the critical exponent, are fully compatible if all possible couplings or derivatives are used.

We interpret this slower transient to the asymptotic regime as an indication of the weakness of the first order transition with antiferromagnetic couplings. This possibility was already conjectured in the case of QCD at finite temperature: indeed the exponent we measure for lattice sizes between $L = 16$ and $L = 20$ is of the order of 2.2, in agreement with the value found from a finite size scaling analysis of the correlation length of QCD Polyakov loops. The results of this paper seem to suggest that such an exponent could be a finite volume effect.

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