THE CRITICAL TEMPERATURE IN RENORMALIZATION GROUP STUDIES OF FIRST ORDER PHASE TRANSITIONS

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We perform a renormalization group study of the first order phase transition of the two dimensional Z(10) Potts model for which an ambiguous determination of the critical temperature was claimed. Allowing the system to flip between the two different phases in the large-volume limit near the critical point one can determine the critical temperature without ambiguity.

Recently renormalization group studies of first order phase transitions have raised a new interest in connection with the determination of the order of the deconfinement transition in lattice quantum chromodynamics. A great effort has been devoted to the study of the three state three dimensional Potts model to which the effective hamiltonian of Polyakov loops of the original four dimensional gauge theory reduces. The model has been studied for different values of the first neighbour and second neighbour couplings and in particular for both ferromagnetic and antiferromagnetic values of the latter. The results obtained with a renormalization group study agree with those obtained from finite size scaling analyses and confirm the first order character of the transition [1]. However, previous renormalization studies of the Z(10) two dimensional model [2] have identified a "pathology" consisting in an ambiguous determination of the discontinuous fixed point of a first order phase transition. According to the renormalization group analysis of Hasenfratz et al., instead of a single value for the critical temperature one obtains two different determinations as a result of the constraint imposed to the system of approaching the criticality from one and only one of the two possible phases coexisting at the transition point. The constraint is implemented by selecting the starting configuration of the system to be in a given phase and by removing the tunneling to the other phase by going to volumes large enough. In these simulations the large-volume limit is performed *before* the limit of a large number of iterations.

In this letter we want to show that by interchanging the order of the two limits one arrives at a unique determination of the critical temperature in agreement with the expected value. The model we have examined is again the Z(10) two dimensional model [3] with first neighbour interactions governed by a coupling β. We have used an update procedure of the Swendsen–Wang–Wolff [4] type in order to let the system to explore efficiently the ten different ground states (colours) of the broken phase. At each Monte Carlo iteration we have identified a cluster of spins connected to a starting spin with a given colour chosen at random on the lattice and we have applied to it the SW algorithm generalised to the case of ten states spins. We have verified the agreement of the results with the standard Metropolis algorithm.

Having many flips of the critical system between the two different phases requires a number of iterations which grows exponentially with the volume. We could then only use moderate volumes, $L=8$, 12, 16 and 24.

We have implemented the finite size real space renormalization group method described in ref. [5] where one studied the renormalization group flow as a function of the total lattice size L for the effective hamiltonian of block variables defined out of site variables belonging to a region of the original lattice which is a *fixed fraction* of L. In previous applications of the method the fraction used was small, i.e. the blocks contained a large number of variables. In the case of strong first order transitions like for the Z(10) model we have learned that by making big blocks one loses the sensitivity of the flow of renormalized couplings to the critical behaviour. We have attributed this fact to the small value of the correlation length in the Z(10) model at criticality where it does not exceed a few lattice spacings: blocks much bigger than the correlation length become uncorrelated and almost insensitive to the underlying critical behaviour. For $L=8$, 12 and 16 we have defined a lattice containing 4×4 block variables obtained by grouping the spin in squares with sides of 2, 3 and 4 lattice spacings respectively. In order to see the volume dependence, we have also made runs with $L=16$ and 24 with 8×8 block variables each. We have made a million iterations with two hundred thousand thermalisation sweeps for $L=8$ and 12 and up to four million iterations with one million thermalisation sweeps for $L=16$ and 24. The probability distribution of the energy and of block variables is expected to be doubly peaked reflecting the coexistence of two phases at criticality: a standard error analysis based on the assumption of variables with a gaussian distribution leads to false estimates.

By grouping the iterations into clusters large enough and defining a single measure as the average over a given cluster one can ultimately recover a gaussian distribution. Our errors have been obtained by comparing runs with hot or cold starts with the statistics quoted above.

We use the majority rule to form block variables out of site variables: we study the flow of the variable C_1 (block energy) defined as

$$C_1 = \sum_{\langle ij\rangle} S_i^B S_j^B , \qquad (1)$$

where S_i^B is the block spin and the sum is over first neighbours only, for various values of L as a function of the original coupling β with the results reported in fig. 1: fig. 1a shows the result of renormalizing by a factor 2, 3 and 4 for $L=8$, 12 and 16 respectively and fig. 1b those of renormalizing by a factor 2 and 3 for $L=16$ and 24. The crossing of the points for different volumes locates the critical coupling whose value converges with increasing L to the expected value: in particular, from the volumes $L=16$ and $L=24$ we estimate

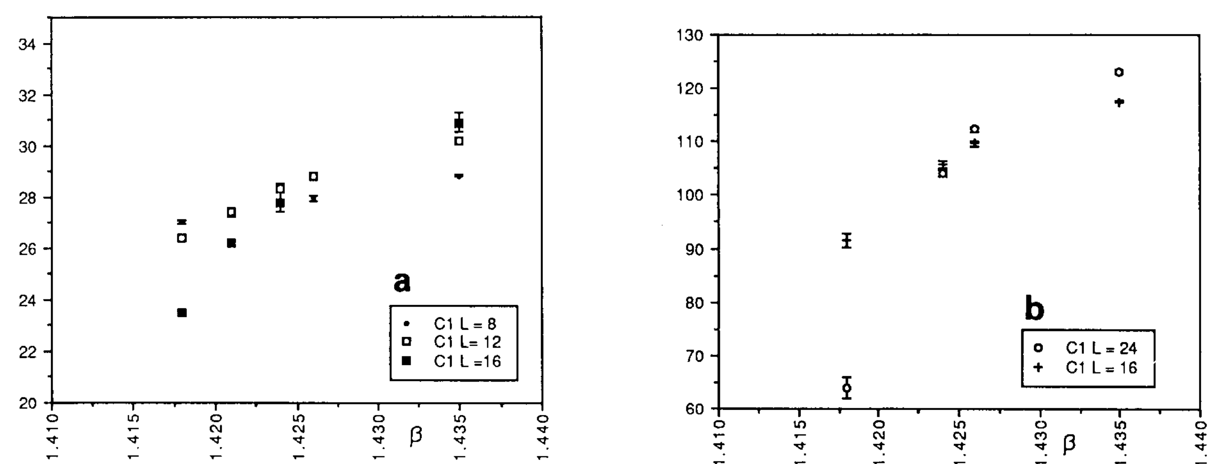

Fig. 1. (a) The flow of coupling C_1 as a function of β obtained by renormalizing by a factor 2, 3 and 4 for $L=8$, 12 and 16 respectively. (b) As in (a) with a renormalization factor 2 and 3 for $L=16$ and 24.

$\beta_c = 1.425(1)$;

we have checked that by using an alternative variable $C_{1\text{-NEXT}}$, defined as the correlation of next to nearest neighbour block spin, one obtains similar results. Through the ratio of the derivatives of the renormalized couplings at the critical point one obtains the estimate of the thermal exponent by the standard formula

$$y_T = \frac{\log[dC_1(L_1)/dC_1(L_2)]}{\log(L_1/L_2)} . \qquad (2)$$

The ratio of derivatives can be estimated either directly by making a linear fit to the coupling's flow near the fixed point or through the derivatives of the block coupling with respect to the original one which can be expressed as the connected correlation of the block coupling with the energy of the original system. By combining both procedures we obtain from the runs at $L=16$ and $L=24$ our best estimate for the thermal exponent averaged in the crossing region of β:

$y_T = 2.1(2)$.

This value is consistent with the one expected for a first order phase transition [6] and with the results of the renormalization group study described in ref. [2].

We have also estimated the magnetic exponent y_h by calculating connected correlations of the block magnetisation with the original one. Their ratio for different values L is equal to the ratio of the derivatives of a magnetic coupling renormalized at different scales which is related to the value of y_h.

In fig. 2 we report the magnetic exponent as a function of β: it reaches a value consistent with two at the transition point. At higher values of β the exponent remains the same indicating the persistence of a first order phase transition in the magnetic field at temperatures lower than the critical one.

We have given evidence that by letting the system oscillate between the two phases of a first order phase transition at any given volume one can determine the transition temperature, without ambiguity, with a renormalization group study of the critical behavior.

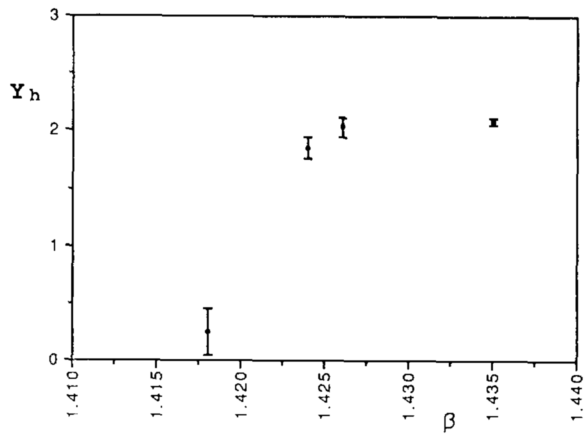

Fig. 2. The magnetic exponent as a function of the temperature.

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